Numerical modeling of a nonlinear resonant vibrometry experiment for crack imaging

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Introduction

What

- **Modeling for resonant laser vibrometry experiment for detecting damage in solids**

  that demonstrates qualitative agreement

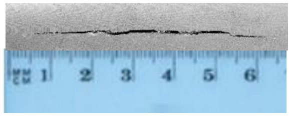
Why

- **Modeling makes all processes “transparent”**

  virtual experiments, full access to simulated data etc

- **Modeling will allow us to estimate defects parameters from measured response**

  and thus completes the NDT strategy



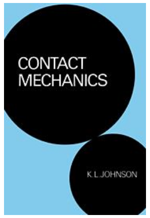
- **Modeling will finally enable to make prognostics**

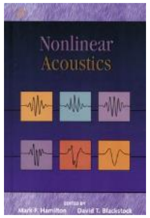
  what happens next, lifetime estimations etc

How

- **Contact model for cracks taking into account friction**

  original Method of Memory Diagrams (MMD)





- **FEM unit for solid mechanics in materials and structures**

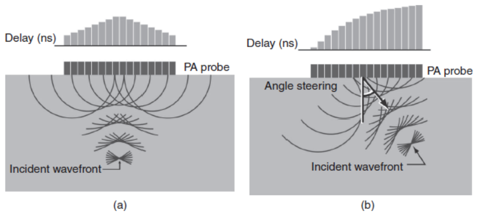
  COMSOL

State of the art: experiment

Dozens of techniques, more than 20 years of development, examples:

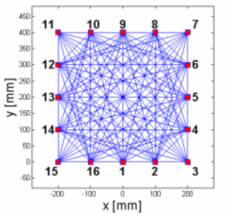
Nonlinear ultrasonic phased array

vary pulse delays, focus at various spots



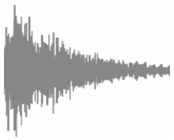
Nonlinear Ultrasonic Guided Wave Tomography

use a set of transducers to generate and record pulses



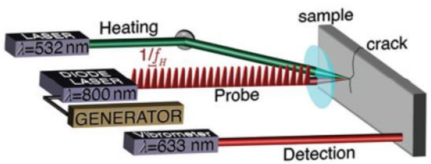
Nonlinear coda wave interferometry

HF coda waves are extremely sensitive to any changes in material



Nonlinear Frequency-Mixing Photoacoustic Imaging

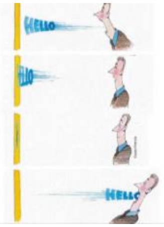
generate acoustic wave via heating by laser, detect by laser



Time reversal

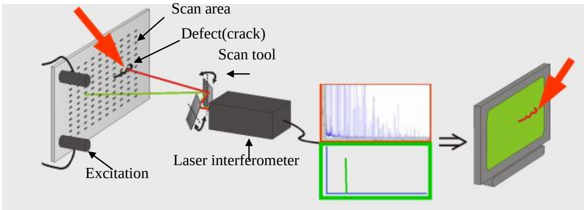
time-reversed signals focus on source

NL time reversed signals focus on damage



Nonlinear resonant scanning laser vibrometry

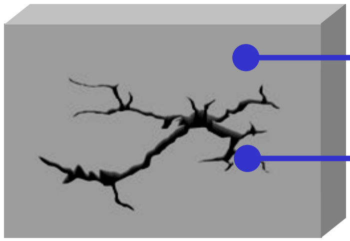
form standing waves, measure harmonics



(+) excitation can be by LF

(-) standing wave is needed

State of the art: friction modeling

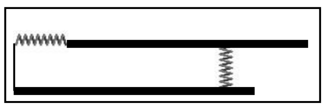


In volume: standard finite elements (FEM)

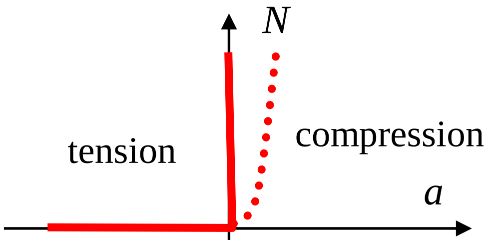
At crack faces: boundary conditions given by

contact model

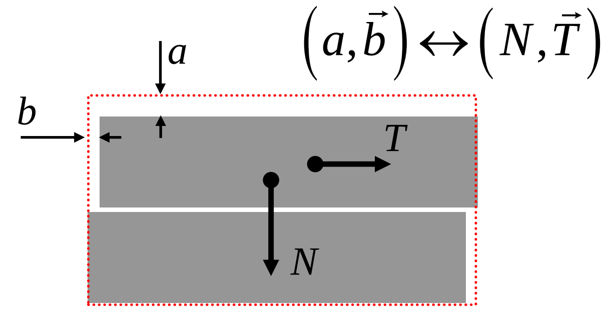
Phenomenological



contains no physics



Plane interface with friction



Coulomb friction law does not provide the boundary condition T(b) explicitly

- If stick then $|T|<\mu N$, b=const
- If slip then $|T|=\mu N$, b unknown

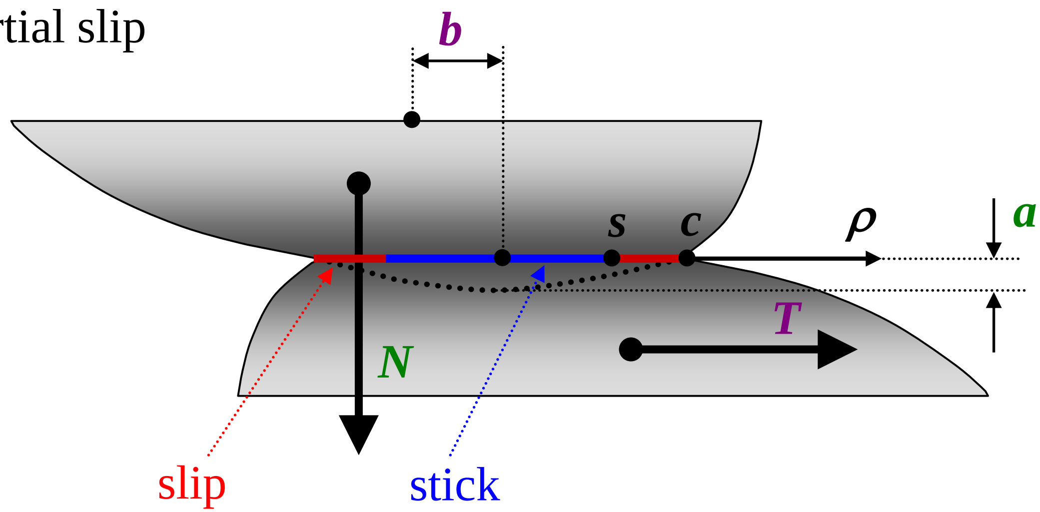
redistribute neighborhood

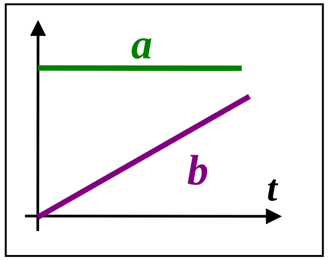
Multiple interrogations of all cells

Implicit calculations

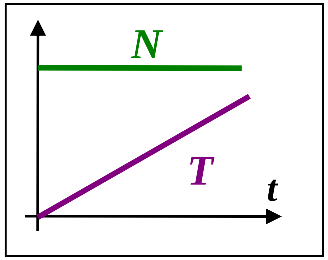
History: Hertz-Mindlin problem

Partial slip





or

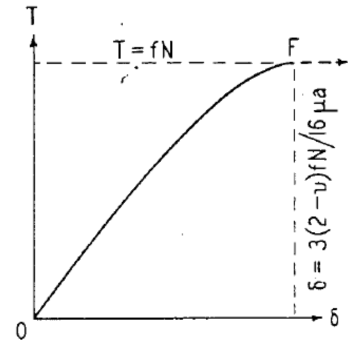


Normal solution: $N = \frac{4E^*}{3R} c^3 \qquad a = \frac{1}{R} c^2$

$$E^* = \frac{E}{2(1-\nu^2)}$$

Tangential solution: $T = \frac{4\mu E^*}{3R}\left(c^3 - s^3\right) \qquad b = \frac{\mu\theta}{R}\left(c^2 - s^2\right)$

$$\theta = \frac{2-\nu}{2(1-\nu)}$$



Mindlin, Deresiewicz (1953)

can be rewritten as

$$\begin{cases} b = \theta\mu(a-q) \\ T = \mu\left(N(a) - N(a=q)\right) \end{cases}$$

with $N = N(a), \quad q = a(c=s)$

Reduced elastic friction principle

J. Jäger (1995, 1998), Ciavarella (1998)

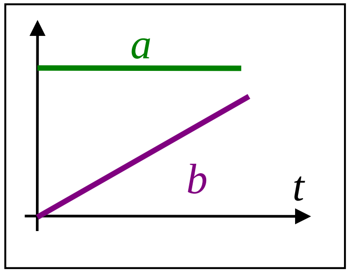
valid for any axisymmetric convex bodies

Tangential = normal – reduced normal

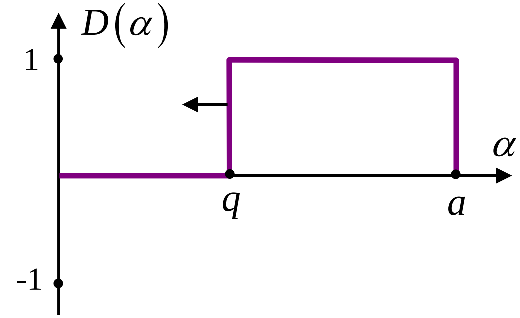
Automate HM mechanics

Method of memory diagrams

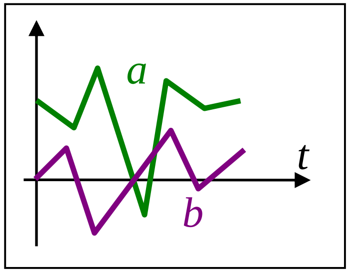
Simple loading in 2D



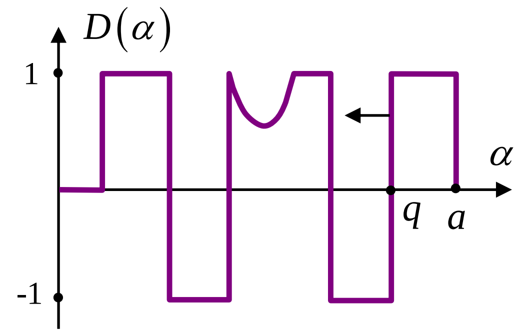
$$\begin{cases} b = \theta\mu\left(a-q\right) \\ T = \mu\left(N\left(a\right) - N\left(a=q\right)\right) \end{cases}$$



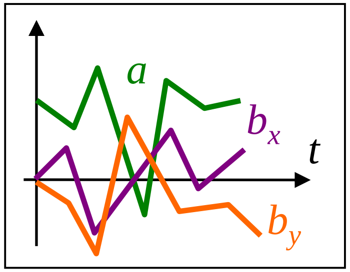
Arbitrary loading in 2D



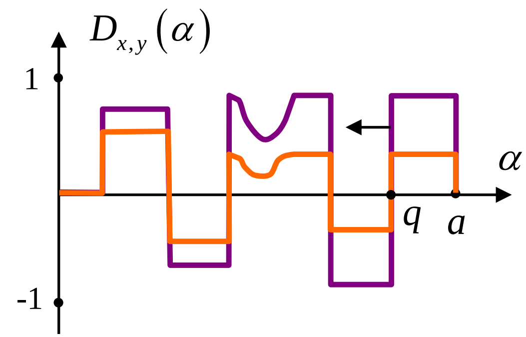
$$\begin{cases} b = \theta\mu\int\limits_0^a D\left(\alpha\right)d\alpha \\ T = \mu\int\limits_0^a D\left(\alpha\right)\left.\dfrac{dN}{da}\right|_{a=\alpha} d\alpha \end{cases}$$



Arbitrary loading in 3D



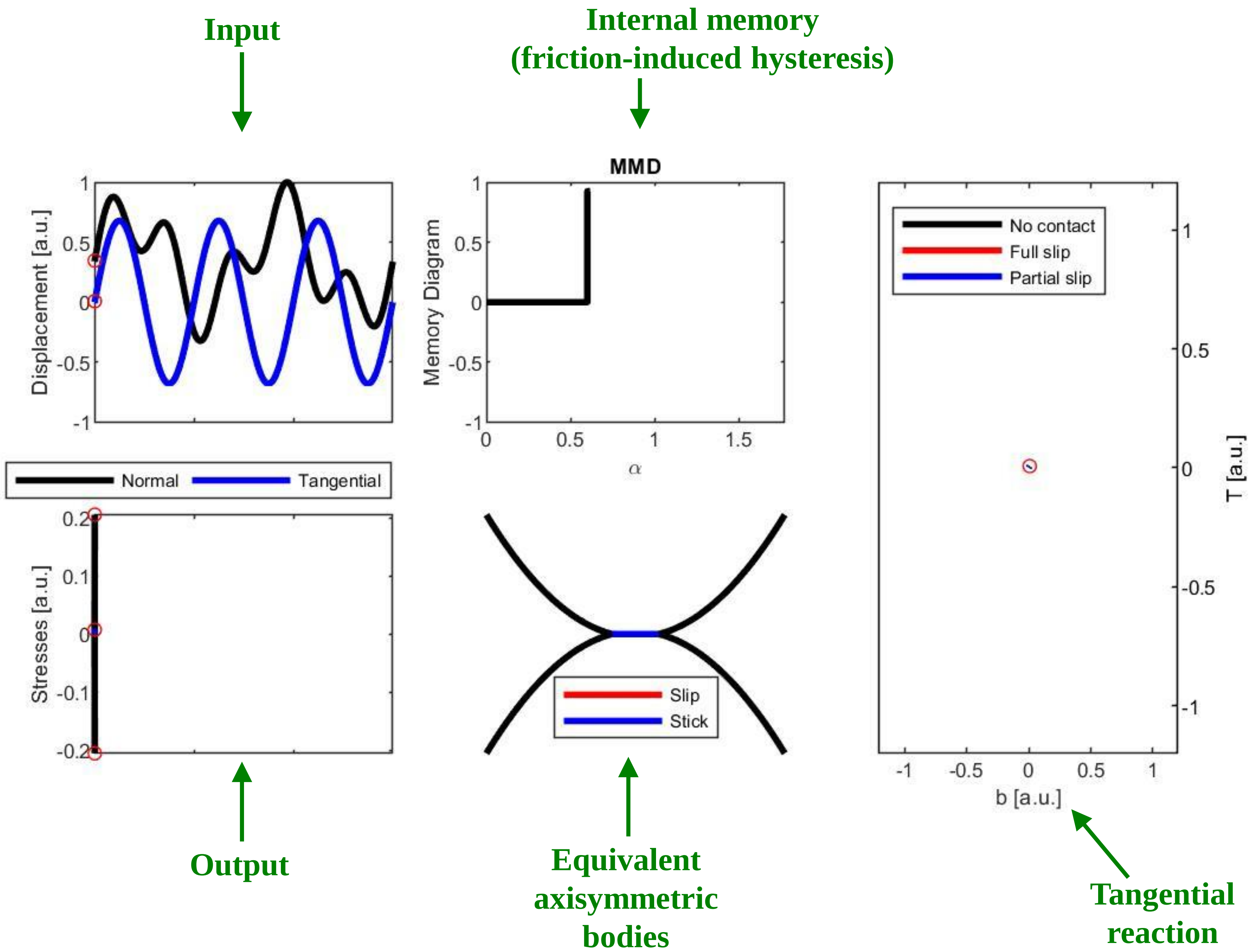
$$\begin{cases} \vec{b} = \theta\mu\int\limits_0^a \vec{D}\left(\alpha\right)d\alpha \\ \vec{T} = \mu\int\limits_0^a \vec{D}\left(\alpha\right)\left.\dfrac{dN}{da}\right|_{a=\alpha} d\alpha \end{cases}$$



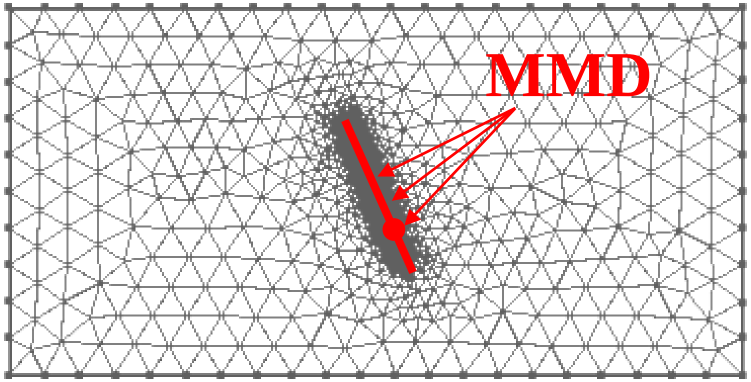
Result: $\vec{T} = MMD\left(\vec{b}\right)$

Consequence: for same normal response same tangential, replace roughness by effective axisymmetric

MMD contact model



MMD-FEM code



Contact model (MMD)

Boundary conditions:

contact displacements and stresses

Solid mechanics module

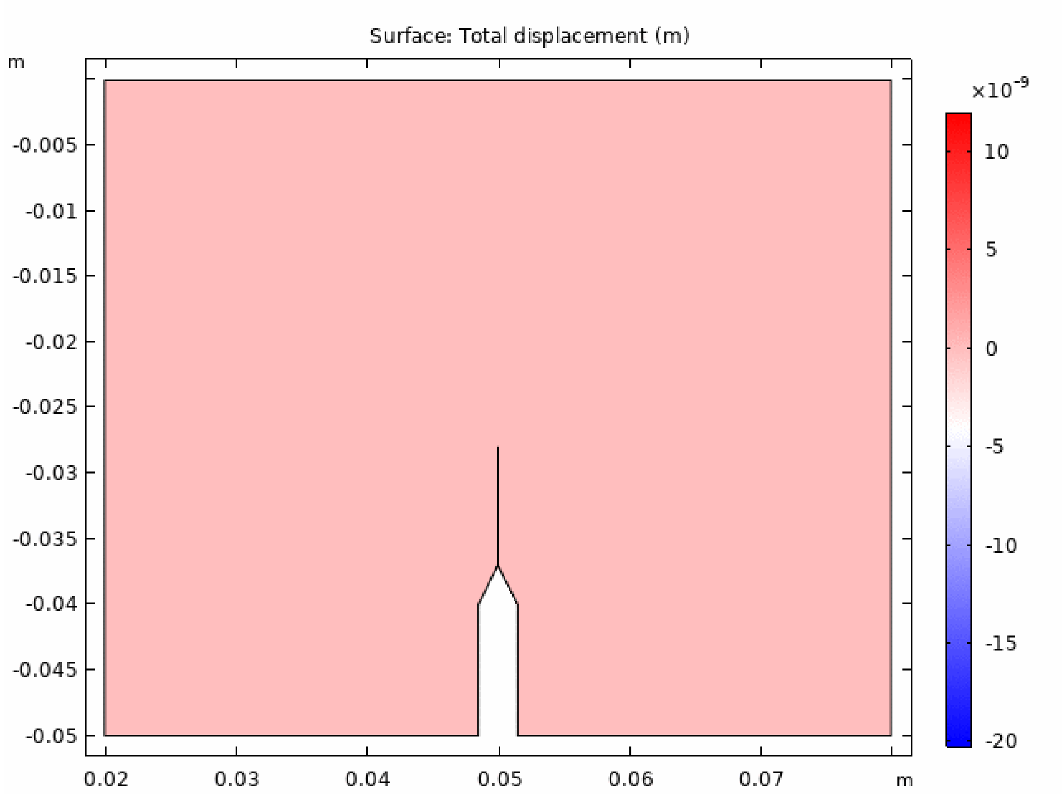
Equations:

bulk displacements and stresses

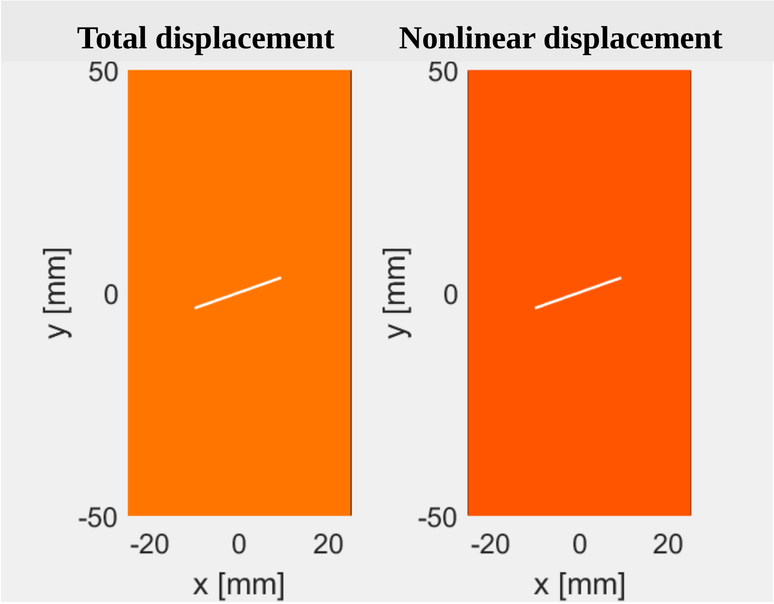
“Thin elastic layer” COMSOL feature

Examples: 2D wave propagation

2D geometry with a notch



Nonlinear secondary waves



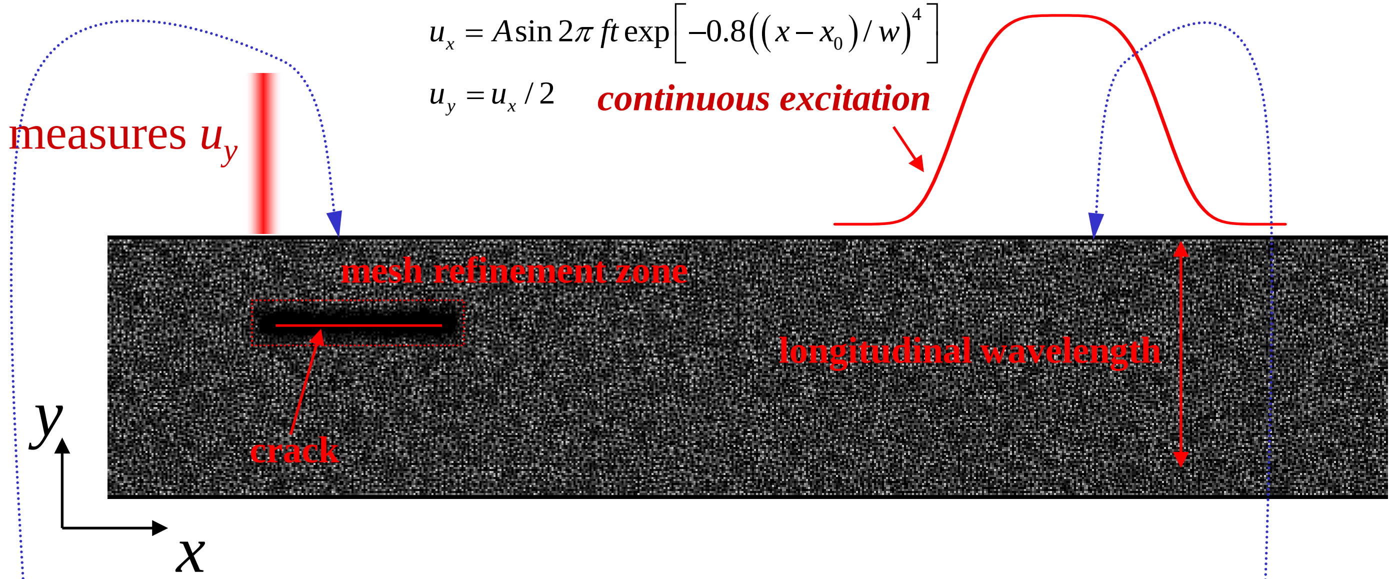
Non-trivial radiation diagram

Conditions do not correspond to any real nonlinear NDT technology

Modeling for resonant vibrometry

$$u_x = A\sin 2\pi ft \exp\left[-0.8\left(\left(x-x_0\right)/w\right)^4\right]$$

$$u_y = u_x/2$$

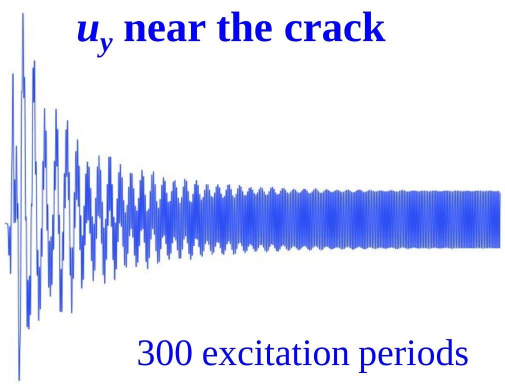


Extremely exaggerated damping (vs typical for metals) in order to build up the standing wave for a reasonable time!

NB: 1 s of acoustic experiment = 10^6 of calculations without crack

5-10 hours in our case with a crack, the crack adds a factor of 5-10

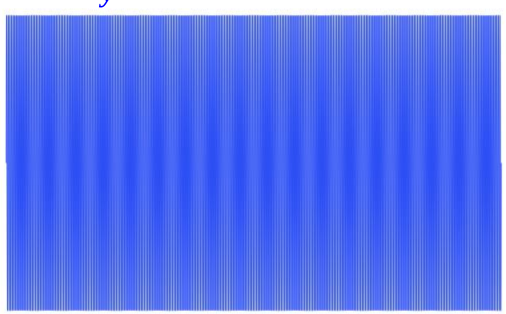
u_y near the crack



300 excitation periods

→ amplitude is 80 times less than →

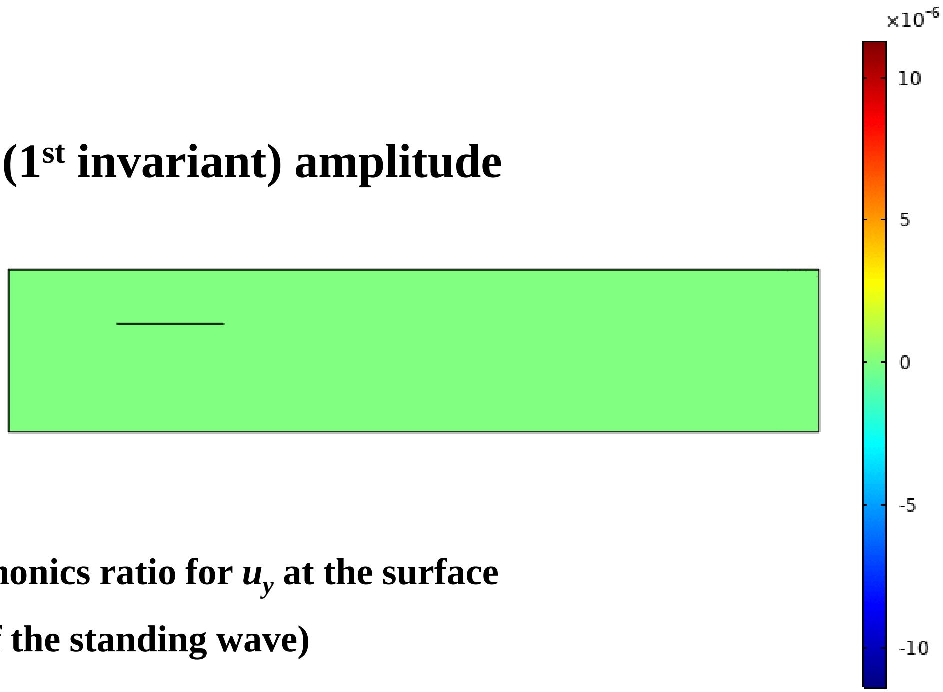
u_y near the transducer

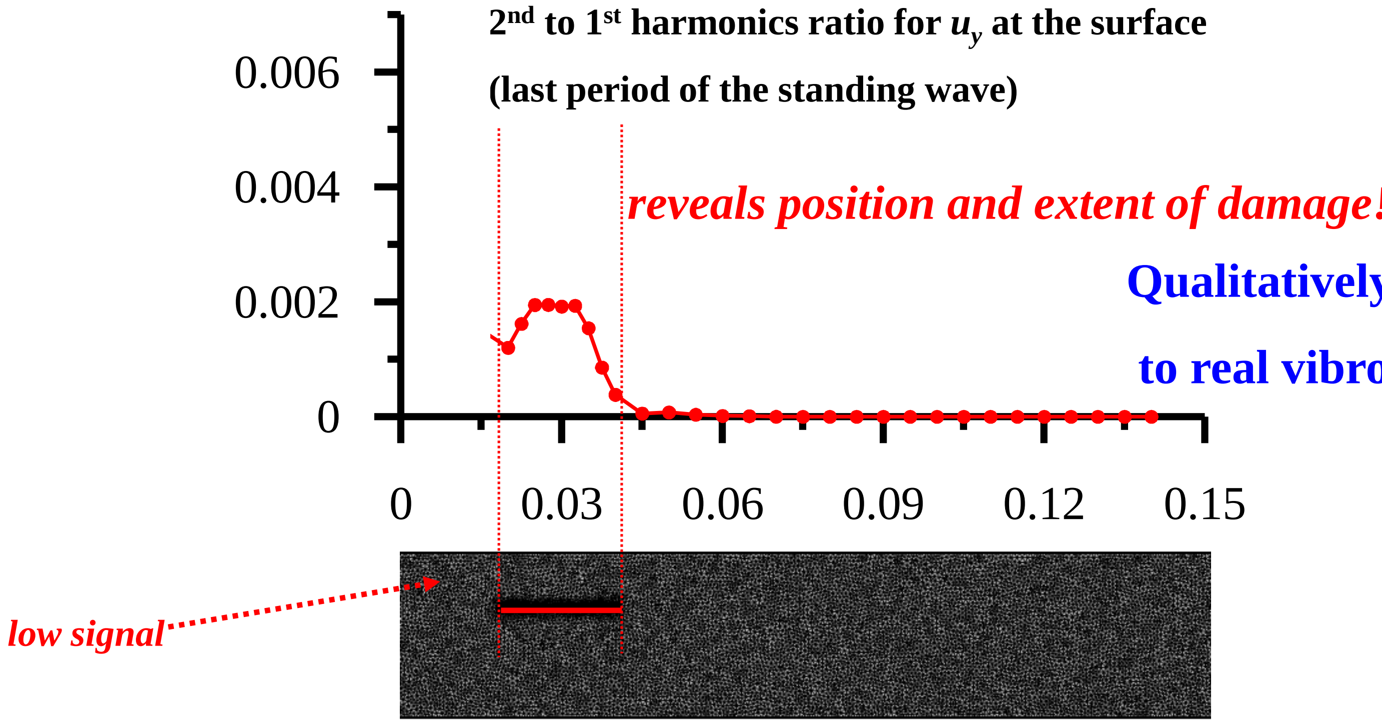


300 excitation periods

Modeling for resonant vibrometry

Strong but realistic strain (1st invariant) amplitude





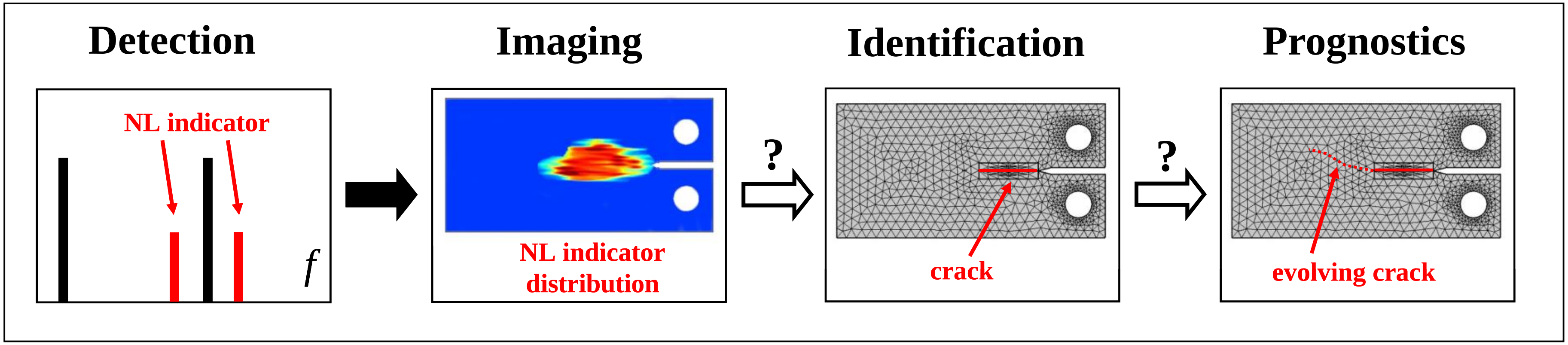
Qualitatively corresponds

to real vibrometry experiments

Conclusions and perspectives

- Gap in parameters between theory and experiment remains
- Qualitative agreement for laser vibrometry experiment
- Seeking for more quantitative agreement

- **NDT applications based on modeling**



- Identification: retrieve information on location, size and orientation of a crack
- Prognostics: use methods of damage mechanics to predict damage evolution

PhD student looking for a postdoc position

Marina TERZI

- PhD expected in February 2022
- Experimental acoustics, focus on NDT
- Numerical acoustics
- Moscow State University graduate

CV_Marina_Terzi_2021.pdf