Application du renversement temporel à la propagation d'ondes haute fréquence dans des assemblages de poutres

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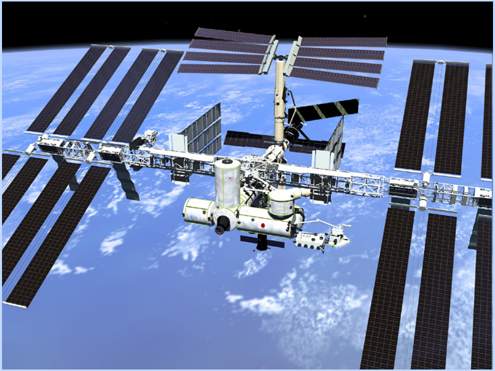
Agenda

Issues

Impulse loads → High frequency (HF) wave propagation

Example: ISS



Agenda

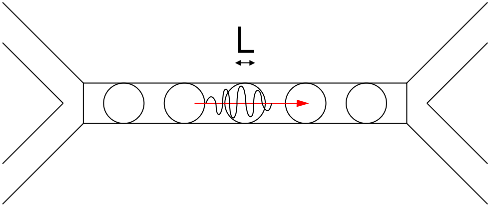
Issues

Impulse loads $\rightarrow$ High frequency (HF) wave propagation

Objectives

- **Develop a model of the transient response of structures subjected to impulse loads;**
- **Use this model in view of time-reversal experiments for defect detection.**

Different models of wave propagation



Modeling scale L	Wavelength	Mean free path	Substructure
Model	Wave equation	Transport	SEA
Typical equation	$\partial_t^2 u - c^2 \partial_x^2 u = 0$	$\partial_t w_\alpha + c \partial_x w_\alpha = 0$	$P_p^{in} = P_p^d + P_p^T$

HF dynamics

- Statistical Energy Analysis (SEA) [Lyon 1975];
- Vibrational Conductivity Analogy (VCA) [Nefske-Sung 1989];
- WKBJ and Gaussian beams [Steele 1976 ... Bougacha 2010];
- Kinetic modeling [Papanicolaou-Ryzhik 1999].

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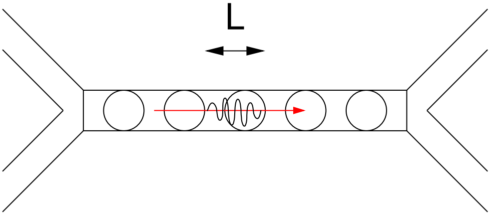

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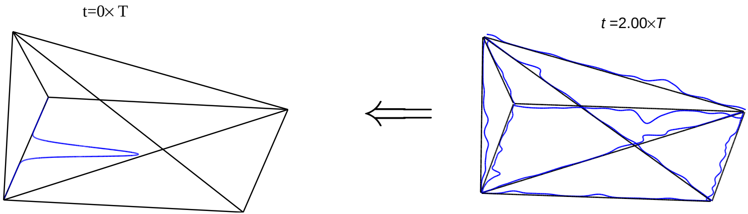
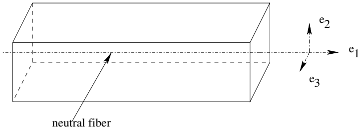
- Transport equation for numerical simulation;
- Vibrational energy density output;
- Diffusive field generated by multiple reflections and transmissions at junctions.



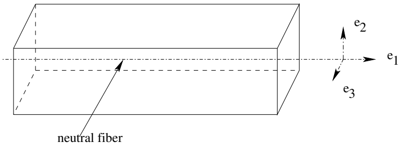
Mechanical hypotheses



Timoshenko beams:

- Piecewise homogeneous material;
- Kinematic assumption: $\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_c(x, t) + \boldsymbol{\theta}(x, t) \times \mathbf{x}_\perp$.

Inside a beam



Transport model:

- 2D version [Savin 2004];
- 3D extension [Le Guennec-Savin 2011].

Dynamic equilibrium for $\mathbf{v} = (\mathbf{u}_c, \boldsymbol{\theta})$

$\Downarrow$

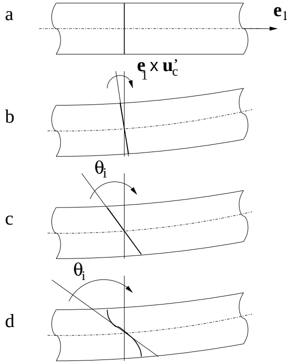
$$H\left(x, \frac{x}{\varepsilon}, \varepsilon\partial_x, \varepsilon\partial_t\right)\mathbf{v}_\varepsilon = 0$$

$\Downarrow$

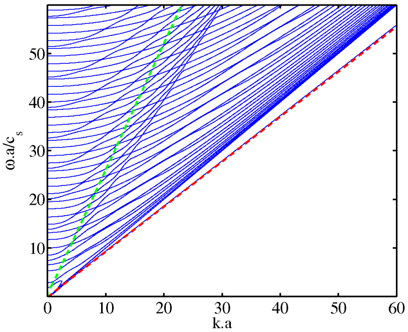
Transport equation for the phase-space energy density as $\varepsilon \to 0$:
$\partial_t w_\alpha + c_\alpha \hat{k}\, \partial_x w_\alpha = 0\,, \quad \alpha \in E\,;$

- $w_\alpha(x,k,t)$: energy density in phase space $\mathcal{S} \times \mathbb{R}_k$ for the mode $\alpha \in E = \{\mathrm{P}_i, \mathrm{T}_i\,, 1 \leq i \leq 3\}$,
- c_α: group velocity of the mode α.

Higher-order kinematics

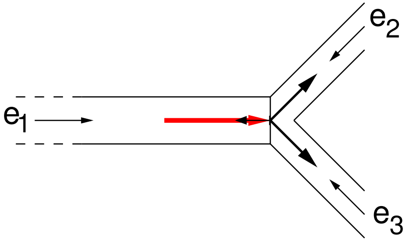


Cross-section kinematics:
(a) reference, (b) Euler-Bernoulli,
(c) Timoshenko, (d) Levinson



Pochhammer-Chree flexural modes
vs. Timoshenko bending/shear
modes

Boundary conditions



Boundary conditions in displacements and efforts for plane waves

⇓

Power flow reflection/transmission coefficients ρ and τ

Numerical ingredients

Reflection/transmission phenomena

⇓

Discontinuity of the energy density field at the junction

⇓

Spectral discontinuous finite element method

+

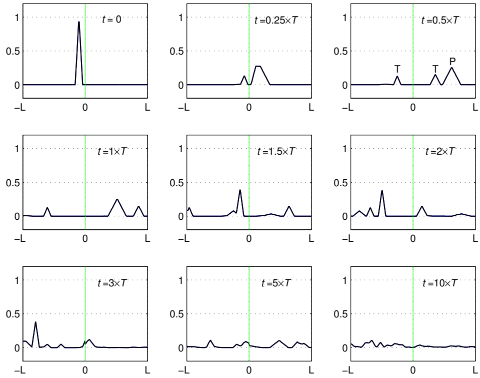
Spatial approximation by Legendre polynomials

+

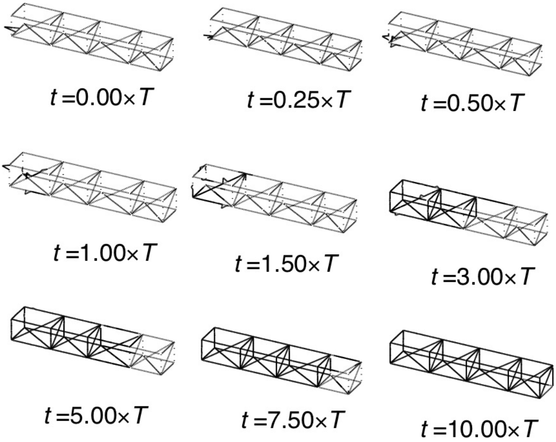
Strong-Stability-Preserving Rung-Kutta (RK-SSP) scheme for time integration

Example

Runge-Kutta discontinuous FEM with Legendre modal expansion and $\mathcal{N} = 40$, $N = 10$, RK-SSP$(8,8)$

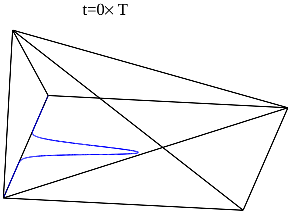


Example

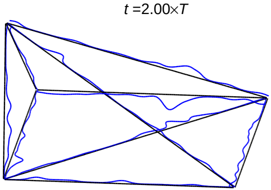


Overview

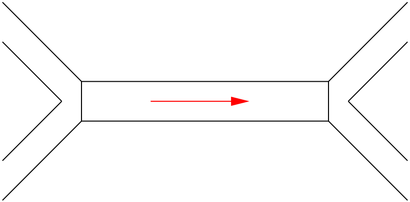
Goal: Recover the initial condition from a diffusive state.



$\Longleftarrow$



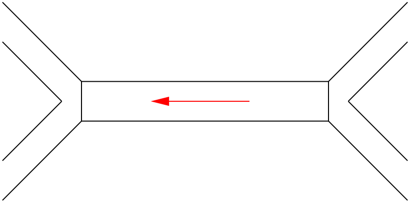
Inside a beam



Direct transport equation:

$$\partial_t w_\alpha^+ + c_\alpha \hat{k} \partial_x w_\alpha^+ = 0$$

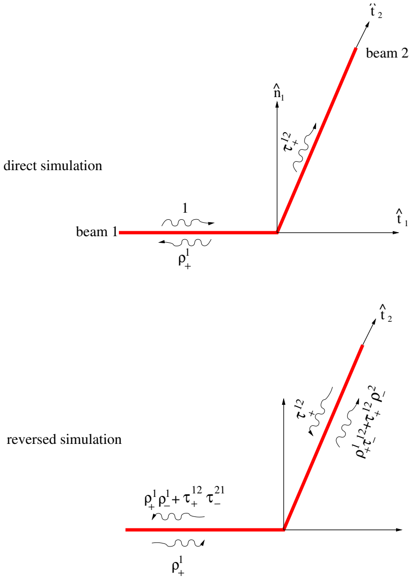
Inside a beam



Reversed transport equation:

$$\partial_t w_\alpha^- + c_\alpha \hat{k} \partial_x w_\alpha^- = 0$$

Time reversal of R/T phenomena

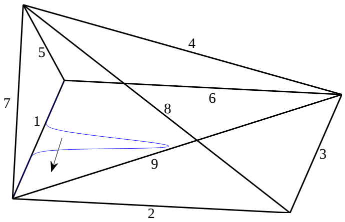


$$\begin{pmatrix} \rho^1_+ & \tau^{12}_+ \\ \tau^{21}_+ & \rho^2_+ \end{pmatrix} \begin{pmatrix} \rho^1_- & \tau^{12}_- \\ \tau^{21}_- & \rho^2_- \end{pmatrix} = \mathbf{I}$$

Example

Numerical parameters:

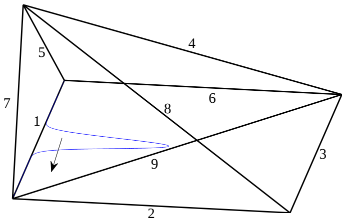
- $\mathcal{N} = 166$ elements for the entire truss;
- Legendre polynomials of order $N = 5$;
- RK-SSP of order 4.



Example

Numerical parameters:

- $\mathcal{N} = 166$ elements for the entire truss;
- Legendre polynomials of order $N = 9$;
- RK-SSP of order 8.



Localization of a defect

Numerical parameters:

- $\mathcal{N} = 98$ elements for the entire truss;
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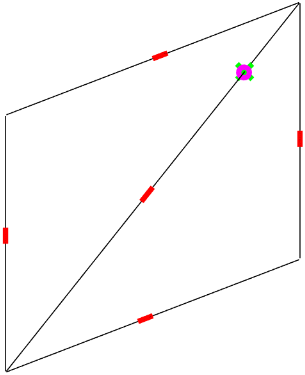

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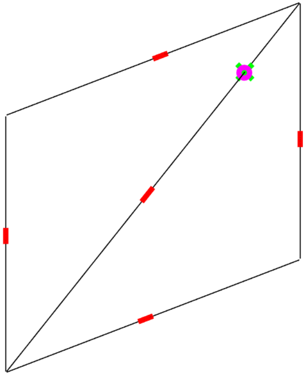

Conclusions

Conclusions:

- Physical model using energy density approach;
- Low dispersion and dissipation numerical scheme;
- Time reversal processing.

Outlook:

- Sensitivity to noise;
- Active sensors;
- Crack detection;
- Use of experimental data.