Quantitative localization of small obstacles in acoustic media.

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[1] Inria Magique3D - UPPA - E2S

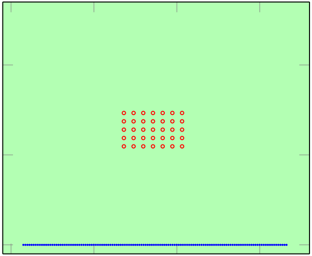
Plan

1. Statement of inverse problem

Motivations

Acoustic wave propagation in complex media program

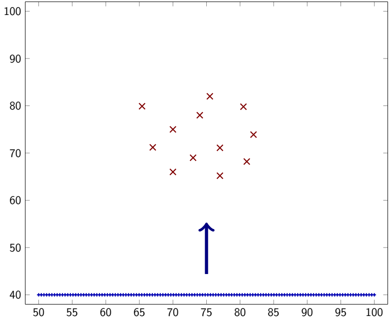
a project piloted by J. Chabassier (Inria)
in collaboration with
acoustic lab I2M (Univ. Bordeaux)



Goal: Efficient numerical tools for

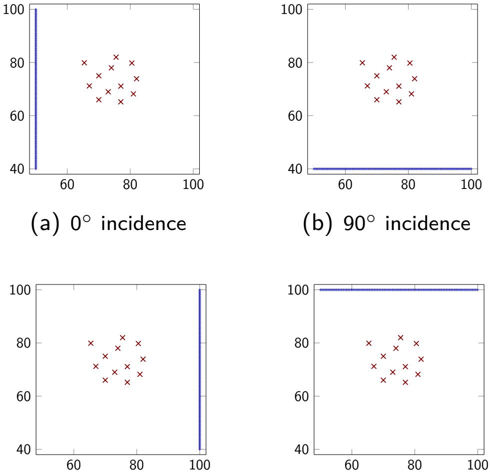
- **direct** modeling and,
- **inverse problem** using **Full-waveform inversion**.

An example of localization problem



90° incidence

An example of localization problem



(a) 0° incidence

(b) 90° incidence

(c) 180° incidence

(d) 270° incidence

- **128 receivers** for each angle of incidence.

Quantitative inversion methodology

For N obstacles, $\mathbf{m} = \left(x_1^{(1)}, x_2^{(1)}, \ldots, x_1^{(N)}, x_2^{(N)}\right)$.

Find the minimizer of the cost function $\mathcal{J}$,

$$\mathcal{J}(\mathbf{m}) = \frac{1}{2} \sum_{t=1}^{N_{\text{Acq}}} \Bigg\| \underbrace{\Phi_t(\mathbf{m})}_{\text{simulated data associated to } \mathbf{m}} - \underbrace{\mathbf{d}_{\mathbf{obs};t}}_{\text{observed data at receptors}} \Bigg\|^2 .$$

Forward map $\Phi_t \; : \; \underset{\text{position of obstacles}}{\mathbf{m}} \longmapsto$ simulated reflected wave recorded at receptors

corresponds to incident wave $u_{\mathbf{inc}}^{(t)}$, $1 \leq t \leq N_{\text{Acq}}$,

Overview

Plan

2. Direct problem (joint with J. Chabassier, H. Barucq, S. Tordeaux)

Multiple obstacle scattering problem

Propagation of acoustic waves of frequency **f** in a medium with sound speed **c**.

$$u_{\textbf{total}} = u_{\textbf{inc}} + u.$$

1. PDE satisfied by u outside the obstacles:

$$(-\Delta - \kappa^2)\, u = 0 \quad , \quad \kappa = \frac{2\,\pi\,\mathbf{f}}{\mathbf{c}}.$$

2. Conditions on the boundary of the obstacles:

$$\begin{array}{lll} \text{Dirichlet} & \gamma_0^+ \; u_{\textbf{total}} & = 0 \\ \text{Neumann} & \gamma_1^+ \; u_{\textbf{total}} & = 0 \end{array}$$

3. Sommerfeld radiation condition at ∞:

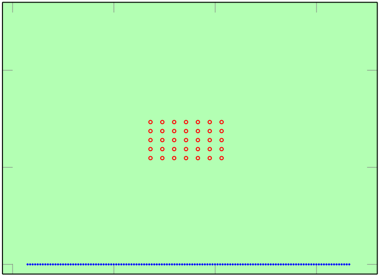
$$\lim_{r\to\infty} \sqrt{r}\,(\partial_r\, u - \mathrm{i}\,\kappa u) = 0 \quad ; \quad r = |x|$$

Time-harmonic Planewave:

$$u_{\textbf{pw}}(x)\ \exp(-i\,2\pi\,\mathbf{f}\,t)$$

$\exists$! solution for the exterior BVPs.

Motivation for direct solver based on single layer potential

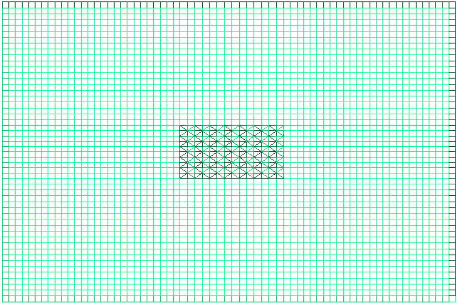


Heterogeneities caused by a large number of small obstacles.

- Domains of size $\geq 100\ \lambda$;
- Obstacle radius $\leq 0.3\ \lambda$.

$\lambda =$ incidence wavelength.

Motivation for direct solver based on single layer potential



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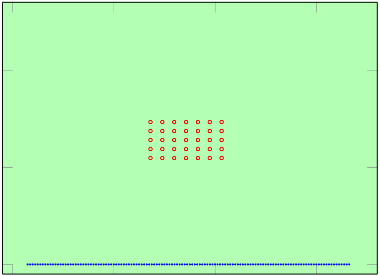

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- Domains of size $\geq 100\ \lambda$;
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Use boundary integral methods

$\rightsquigarrow$ **single-layer potential**.

Fourier Series Single Layer method (FSSL)

Step 1 : With N obstacles, the scattered wave is

$$u = \sum_{J=1}^{N} \underbrace{u_J}_{\text{wave scattered by obs. J}} \quad ;$$

$$u_h = \sum_{J=1}^{N} \underbrace{u_{h;J}}_{\text{approximate scattered wave by obs. J}} \quad .$$

Fourier Series Single Layer method (FSSL)

Step 1 : With N obstacles, the scattered wave is

$$u = \sum_{J=1}^{N} \underbrace{u_J}_{\text{wave scattered by obs. J}} = \sum_{J=1}^{N} \mathcal{S}_J \, v_J;$$

$$u_h = \sum_{J=1}^{N} \underbrace{u_{h;J}}_{\text{approximate scattered wave by obs. J}} = \sum_{J=1}^{N} \mathcal{S}_J \, v_{h,J}.$$

$\rightsquigarrow$ Satisfies the outgoing condition.

Single-layer potential on obstacle J with **density** ϕ

$$(\mathcal{S}_J \, \phi)(x) = \int_{\Gamma_J} \frac{\mathrm{i}}{4} \, \mathrm{H}_0^{(1)}(\kappa|x-y|) \, \phi(y) \, ds(y).$$

FSSL (cnt)

Step 2 : Discretization of the densities

Fourier basis on the boundary Γ_J of obstacle J

$$\mathbf{w}_{J,k}(\gamma_J(\theta)) = e^{i\,k\,\theta} \quad , \quad \gamma_J : [0, 2\pi)_\theta \to \Gamma_J .$$

$$u_J = \sum_{J=1}^{N} \mathcal{S}_J \, v_J \qquad ;$$

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$$u_{h;J} = \sum_{J=1}^{N} \mathcal{S}_J \boxed{\sum_{k=-\mathbf{m}}^{\mathbf{m}} V_{J,k}\; \mathbf{w}_{J,k}} \qquad .$$

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$$(\mathcal{S}_J\;\mathbf{w}_{J,k})(x) \;=\; \int_{\Gamma_J} \tfrac{\mathrm{i}}{4}\;\mathrm{H}_0^{(1)}(\kappa|x-y|)\;\mathbf{w}_{J,k}(y)\,ds(y)$$

$$=\int_0^{2\pi} \tfrac{\mathrm{i}}{4}\;\mathrm{H}_0^{(1)}(\kappa|x-\gamma_J(\theta)|)\;e^{\mathrm{i}\,k\,\theta}\;|\gamma_J'(\theta)|\,d\theta\,.$$

FSSL (cnt)

Step 3 : Impose boundary conditions to obtain linear system.

$$\mathbf{A}\, V = F \quad , \quad \mathbf{A}_h\, V_h = F_h\,.$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1(\mathbf{N}-1)} & \mathbf{A}_{1\mathbf{N}} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2(\mathbf{N}-1)} & \mathbf{A}_{2\mathbf{N}} \\ \vdots & \cdots & \ddots & \cdots & \vdots \\ \mathbf{A}_{(\mathbf{N}-1)1} & \mathbf{A}_{(\mathbf{N}-1)2} & \cdots & \mathbf{A}_{(\mathbf{N}-1)(\mathbf{N}-1)} & \mathbf{A}_{(\mathbf{N}-1)\mathbf{N}} \\ \mathbf{A}_{\mathbf{N}1} & \mathbf{A}_{\mathbf{N}2} & \cdots & \mathbf{A}_{\mathbf{N}(\mathbf{N}-1)} & \mathbf{A}_{\mathbf{NN}} \end{pmatrix}$$

$\mathbf{A}_I$ self-interaction of obstacle I

$\mathbf{A}_{IJ}$ diffraction by obs. I of wave emitted by obs. J

Cropped from $\mathbf{A}$, matrix $\mathbf{A}_h$ is of size $[(2\mathbf{m}+1)\times N]^2$,

FSSL with circular obstacles with radius r_I.

Basis of scattered wave in multipole expansions

Single-layer potential with density $\mathbf{w}_{J,k}$

$$(\mathcal{S}_J\,\mathbf{w}_{J,k})\,(x) = \frac{i\pi\, r_J}{2}\, J_k(\kappa\, r_J) \qquad \underbrace{H_k^{(1)}(\kappa\, r_J(x))\; e^{i\,k\,\theta_J(x)}}_{\text{multiple pole of order k placed at the center of } \mathcal{O}_J} \qquad .$$

Components of matrix A in multipole expansions

Same obstacle interaction

$$(\mathbf{A}_I)_{kl} \;=\; i\pi\, r_I\; J_k(\kappa\, r_I)\; \delta_{kl} \begin{cases} H_k^{(1)}(\kappa\, r_I) & \text{Dirichlet} \\ \kappa\; H_k^{(1)\prime}(\kappa\, r_I) & \text{Neumann} \end{cases} \quad , \quad k, l \in \mathbb{Z}\,.$$

Interaction between two different obstacles $I \neq J$

$$(\mathbf{A}_{IJ})_{kl} = i\,\pi\, r_J\, e^{i(l-k)\theta_{\mathbf{x}_J}(\mathbf{x}_I)}\; H_{l-k}^{(1)}\,(\kappa\,|\mathbf{x}_I - \mathbf{x}_J|)\; J_k(\kappa\; r_I) \begin{cases} J_l(\kappa\, r_J) & \text{Dirichlet} \\ \kappa\, J_l'(\kappa r_I) & \text{Neumann} \end{cases} ,$$

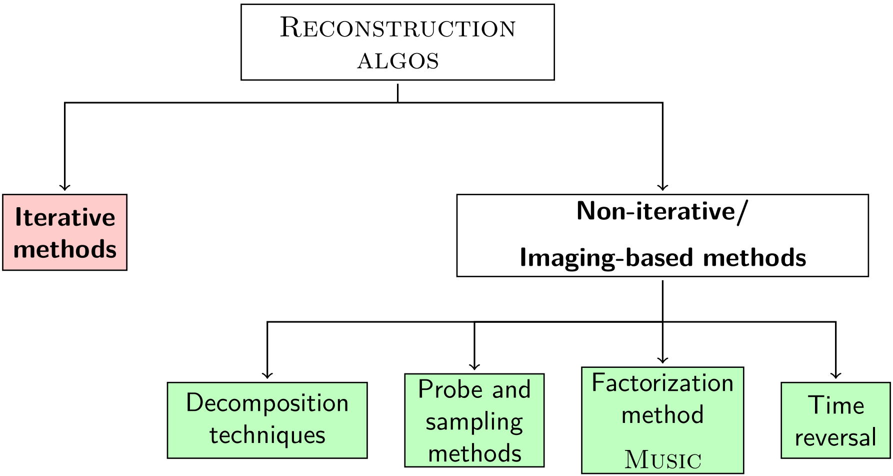
Relative polar coordinates $(r_J(\cdot), \theta_J(\cdot))$ with respect to obstacle $\mathbf{x}_J$

$$x = \mathbf{x}_J + r_J(x)(\cos\theta_J(x), \sin\theta_J(x))$$

Plan

3 Inverse problem - FWI

Literature of inversion methods



Inversion methodology main features

Find the minimizer $\mathcal{J}(\mathbf{m}) = \frac{1}{2} \sum_{t=1}^{N_{\text{Acq}}} \| \Phi_t(\mathbf{m}) - \mathbf{d}_{\mathbf{obs};t} \|^2$.

Line-search optimization strategy

$$\mathbf{m}_{i+1} = \mathbf{m}_i + \underbrace{\alpha_i}_{\text{step length}} \underbrace{\mathbf{s}_i}_{\text{search direction}} .$$

- **Search direction** $\mathbf{s}_i$ given by Quasi-Newton BFGS or, Nonlinear conjugate gradient Polak-Ribière.
- Step length α_i is given by **line-search algorithm** Simple back-tracking, sufficient descent, or Strong Wolfe criteria

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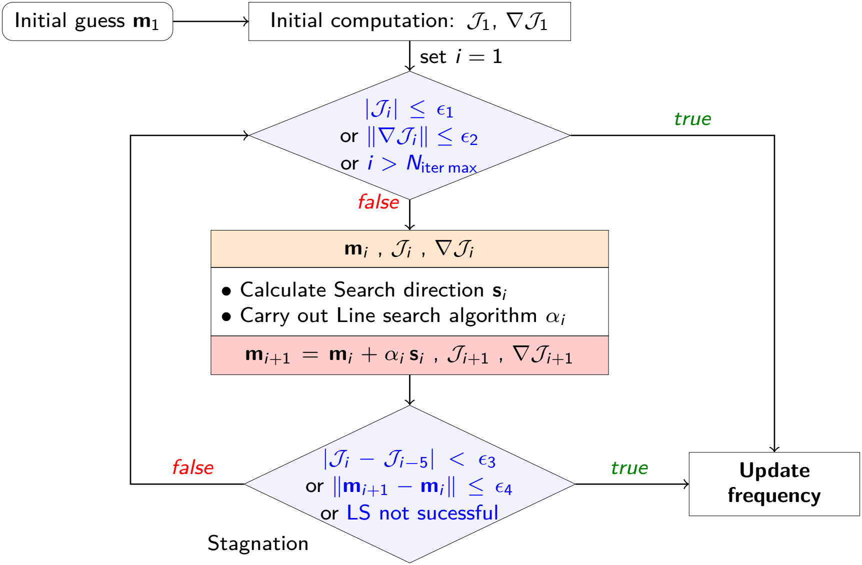
Frequency-hopping and recycling to escape from stagnation in local minima.

Optimization algorithm at a frequency



Plan

4 Adjoint state method

Adjoint method for calculating the gradient

$$\mathcal{J}'(\mathbf{m}) = \sum_{\mathrm{t}=1}^{N_{\mathsf{Acq}}} \mathrm{Re}\left((\Phi_{\mathrm{t}} - \mathbf{d}_{\mathrm{t}})^{\star} \, \partial_{\mathbf{m}} \Phi_{\mathrm{t}} \right).$$

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Advantage of using the adjoint method

applied to the discrete problem

- Avoid calculating the Jacobian $\partial_{\mathbf{m}}\Phi$
- Avoid calculating $\partial_{\mathbf{m}}\mathbf{A}^{-1}$.

$$\textbf{Forward map } \Phi \quad : \quad \text{Model space} \longrightarrow \begin{array}{c}\text{Simulated}\\ \text{data space}\end{array}$$
$$\mathbf{m} \quad \mapsto \quad u_{h,\text{scatt}}\big|_{\text{receivers}}$$

$$u_{h,\textbf{scatt}}(x) \; = \; T(\mathbf{m}) \underset{(J,l)}{\cdot} V(\mathbf{m}) \; = \; \sum_{J=1}^{N_{\text{Obs}}} \sum_{l=-\text{n}}^{\text{n}} V_{J,l} \; T(\mathbf{m})_{J,l} \, .$$

- coefficients $V_{J,l}$ solves the scattering linear system, and
- $$T(\mathbf{m})_{J,l} \; = \; \underset{\text{single-layer potential}}{\mathcal{S}_J \, \mathbf{w}_{J,l}} \; = \; \frac{\mathrm{i}\,\pi \; \mathrm{r}_J \; \mathrm{J}_l(\kappa \, \mathrm{r}_J)}{2} \; \mathrm{H}_l^{(1)}(\kappa \, r_J(x)) \; e^{\mathrm{i}\, l\, \theta_J(x)} \quad .$$

$$\Phi(\mathbf{m}) \; = \; u_{h,\text{scatt}}\big|_{\text{receivers}} \; = \; T(\mathbf{m})^t\big|_{\text{receivers}} \; V(\mathbf{m}) \; = \; \mathfrak{R}(\mathbf{m}) \; V(\mathbf{m}).$$

Adjoint method for calculating the gradient (cnt)

Step 1 : Solve forward problem ($\rightsquigarrow$ obtain $\mathcal{J}$ at $\mathbf{m}$).

$$\mathbf{A}(\mathbf{m}) \left[\mathsf{S}_1 \quad \ldots \quad \mathsf{S}_{N_{\text{Acq}}} \right] = \left[F(\mathbf{m}, u_{\text{pw}}^{(1)}) \quad \ldots \quad F(\mathbf{m}, u_{\text{pw}}^{(N_{\text{Acq}})}) \right].$$

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Step 2: Solve the adjoint equation for adjoint states

$$\mathbf{A}(\mathbf{m})^{\star}\left[\gamma_1(\mathbf{m}) \quad \dots \quad \gamma_{N_{\text{Acq}}}(\mathbf{m})\right]$$

$$= -\mathfrak{R}^{\star}(\mathbf{m})\left[\ (\Phi_1(\mathbf{m}) - \mathbf{d}_1) \ \dots \ (\Phi_{N_{\text{Acq}}}(\mathbf{m}) - \mathbf{d}_{N_{\text{Acq}}})\ \right].$$

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Step 3 The derivative of the cost function $\mathcal{J}$ at $\mathbf{m}$

$$\mathcal{J}'(\mathbf{m}) = \sum_{\mathrm{t}=1}^{N_{\mathrm{Acq}}} \mathrm{Re}\Big[\; (\Phi_{\mathrm{t}}(\mathbf{m}) - \mathbf{d}_{\mathrm{t}})^{\star} \;\; \partial_{\mathbf{m}}\mathfrak{R} \underset{(J,l)}{\cdot} \mathrm{S}_{\mathrm{t}}(\mathbf{m}) + \gamma_{\mathrm{t}}{}^{\star} \Big(\partial_{\mathbf{m}}\mathbf{A} \underset{(J,l)}{\cdot} \mathrm{S}_{\mathrm{t}}(\mathbf{m}) \; - \; \partial_{\mathbf{m}} F(\mathbf{m}, u_{\mathrm{pw}}^{(\mathrm{t})}) \Big) \;\Big].$$

Plan

5 Optimization method comparison Experiment with 6 obstacles

Inexact Line search algorithm

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \overset{?}{\alpha_k} \, \mathbf{s}_k .$$

A **line-search algorithm** produces an approximate minimum of

$$\min_{\alpha > 0} \phi(\alpha) \quad ; \quad \phi(\alpha) = \mathcal{J}\left(\mathbf{m}_k + \alpha \, \mathbf{s}_k\right)$$

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LS 1

- Simple back-tracking criteria

$$\phi(\alpha) < \phi(0)$$

$\star$ Update with

$$\alpha \mapsto \frac{1}{2} \alpha .$$

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LS 2

- Sufficient decrease (Armijo) condition

$$\phi(\alpha) < \phi(0) + c_1\, \alpha\, \phi'(0)\, ,$$

$$c_1 \in (0, 1).$$

⋆ Update with

$\alpha \mapsto$ quadratic interpolation on $[0,\alpha]$

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$$c_1 \in (0, 1).$$

⋆ Update with

$$\alpha \mapsto \underset{\text{interpolation on } [0,\alpha]}{\text{quadratic}}$$

LS 3 : Strong Wolfe

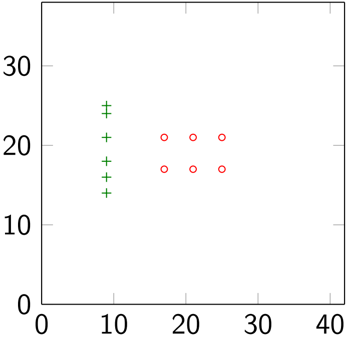
- Armijo condition.
- Curvature condition

$$|\phi'(\alpha)| \leq c_2 |\phi'(0)| ,$$

$$0 < c_1 < c_2 .$$

⋆ Search interval update by *Zoom* algo. (Nocedal).

Configuration with 6 obstacles using 23dB data



∘ = True positions .

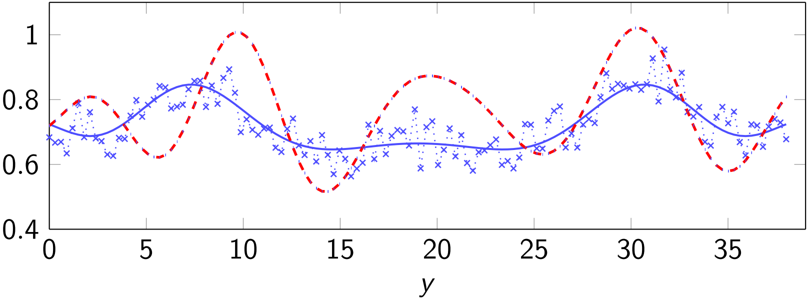
+ = Initial guess 1.

Error Position = 20.5.

Relative Error Position = 54%.

Configuration with 6 obstacles using 23dB data

23dB $\rightsquigarrow$ relative L^2 error between 6% and 8%.



Real part of the data at 128 receivers for incidence angle 0° and freq $\kappa = 1$.

(*Relative error in* L^2*-norm, between observed and noisy data, is* 7%).

$\cdots\times\cdots$ = true synthetic data ; —— = true synthetic data with 23dB noise;

- - - = simulated data by initial guess 1.

Results with 200 randomly generated IG-s using 4 incidence angles.

Method	SD1-LS1	SD1-LS2	SD1-LS3	SD2-LS1	SD2-LS2	SD2-LS3
Success rate	91%	83%	99%	85%	75%	92%
Avg. run time (s)	7.6	2.1	2.0	16.0	4.3	2.8
Std. dev. run time (s)	1.1	0.7	0.5	6.0	1.4	0.6
Avg. iter number	221	291	150	417	655	234
Std. dev. iter number	39	114	31	149	237	45
Avg final $\mathcal{J}$	3×10^{-10}	5×10^{-11}	5×10^{-11}	7×10^{-9}	8×10^{-8}	7×10^{-11}
Std. dev. final $\mathcal{J}$	6×10^{-10}	1×10^{-10}	2×10^{-11}	2×10^{-8}	5×10^{-7}	9×10^{-11}

Observations

- Choice of line search algorithm option has more influence.

⊛ There is a clear benefit in using the LS3 option, with the best success rate for both search directions.

- The most robust and effective method in this configuration are SD1-LS3, and SD2-LS3 slightly behind.

- LS1 is the simplest method to implement, however is most time consuming in applications.

Plan

6 Reconstruction with 12 obstacles

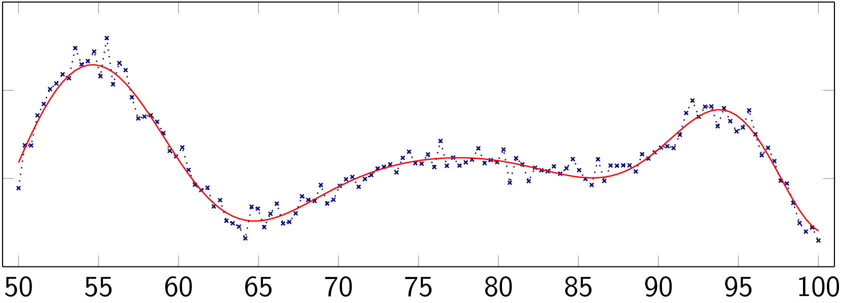
12 hard-scattering obstacles reconstruction

- **128 receivers** for each angle of incidence.
- **Four angles** of back-scattered data acquisitions

12 hard-scattering obstacles reconstruction

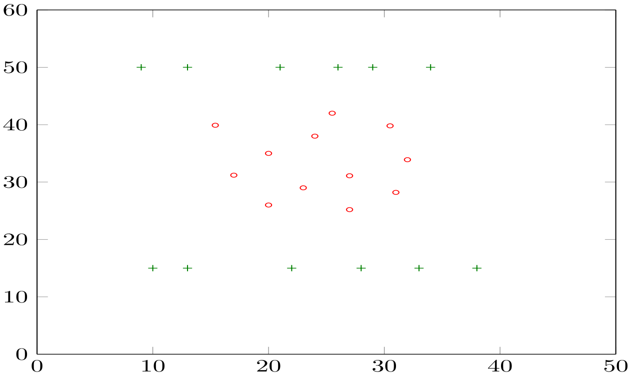
- **128 receivers** for each angle of incidence.
- **Four angles** of back-scattered data acquisitions
- **Noisy synthetic data at 30dB**

⇝ Relative L^2 error between 2.7% and 3.6%.



Locating 12 hard-scattering obstacles results

- Quasi-Newton method with Strong Wolfe linesearch,
  - 10 frequencies with $0.08 \leq$ wavenumber ≤ 0.8 .



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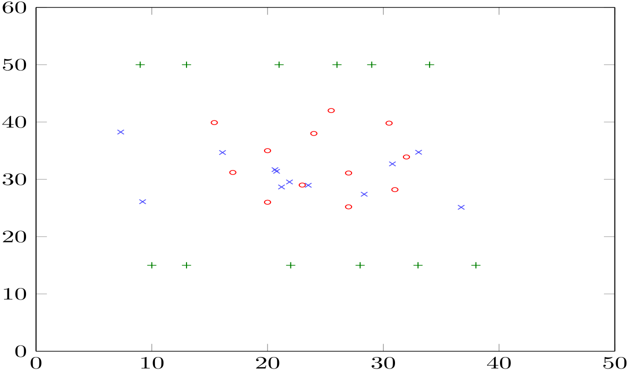

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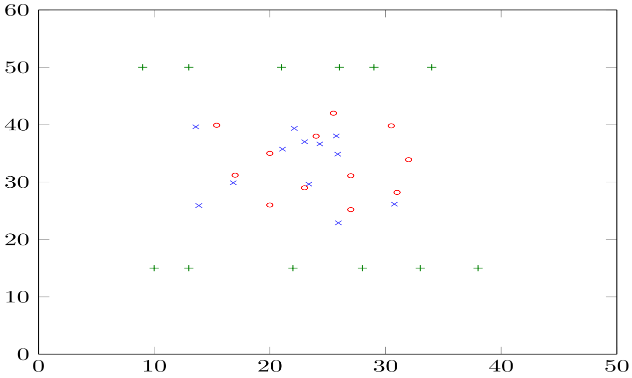

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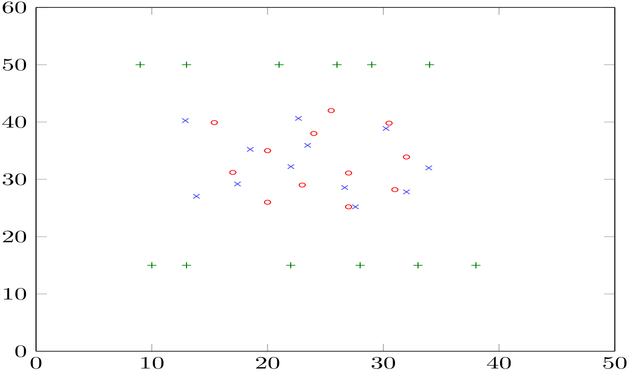

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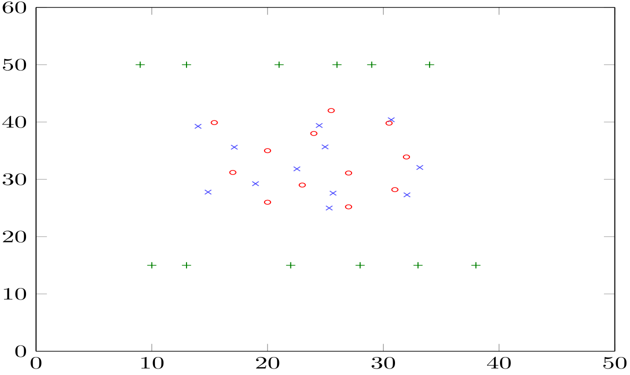

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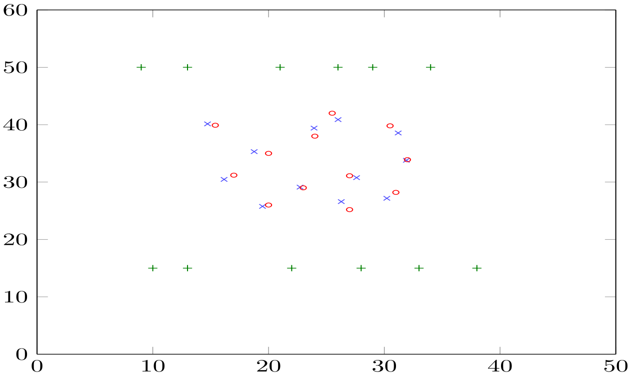
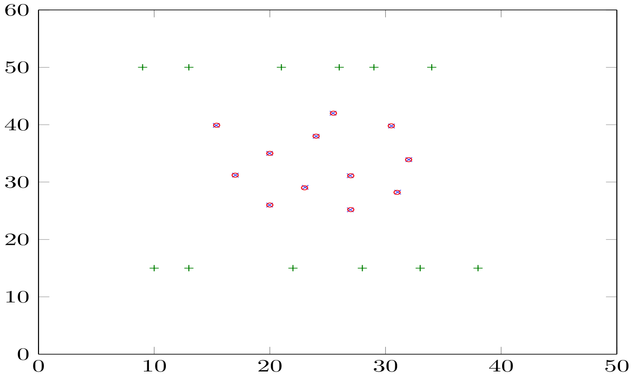
- Quasi-Newton method with Strong Wolfe linesearch,
- 10 frequencies with $0.08 \leq$ wavenumber ≤ 0.8 .



Locating 12 hard-scattering obstacles results

- 317 iterations
- Final relative position error = 0.3%.

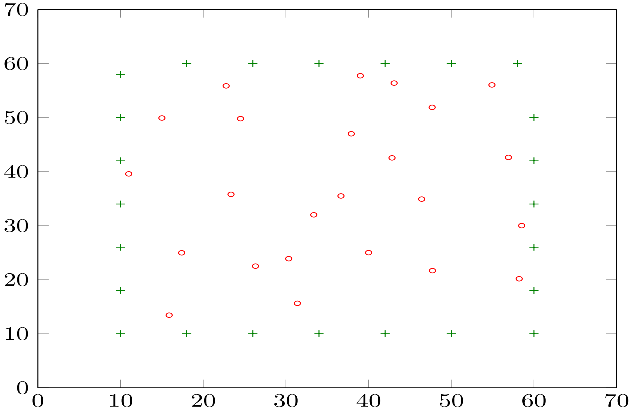
Run time = 7.7s.



Plan

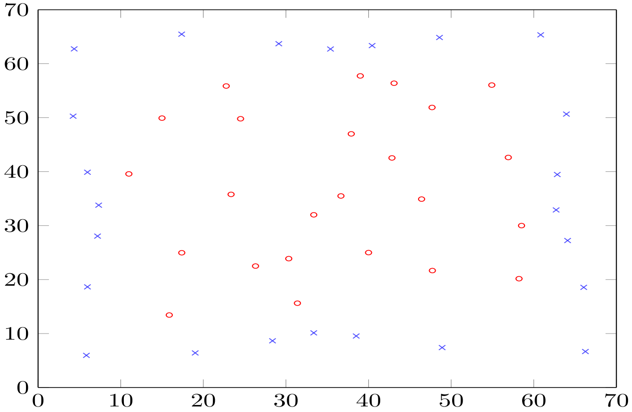
7 Reconstruction with 24 obstacles

Locating 24 obstacles with noise-free data



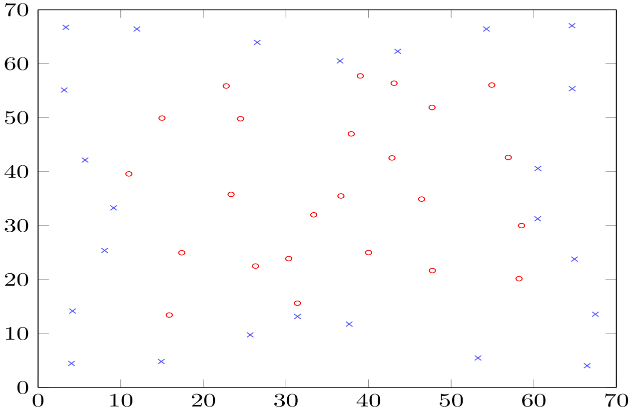
Total run time : 260 s

Locating 24 obstacles with noise-free data



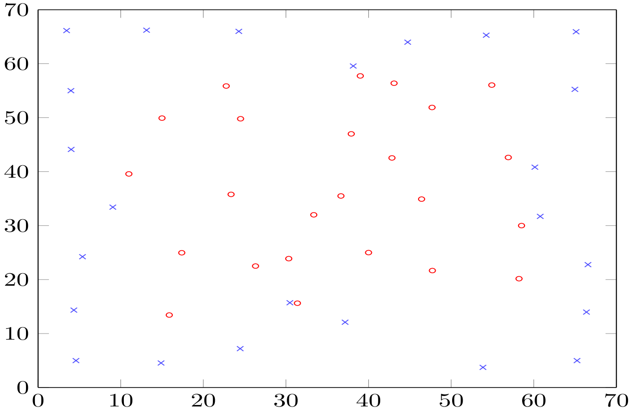
Total run time : 260 s

Locating 24 obstacles with noise-free data



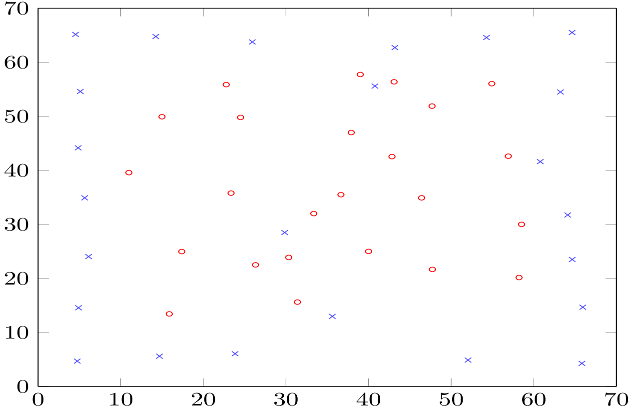
Total run time : 260 s

Locating 24 obstacles with noise-free data



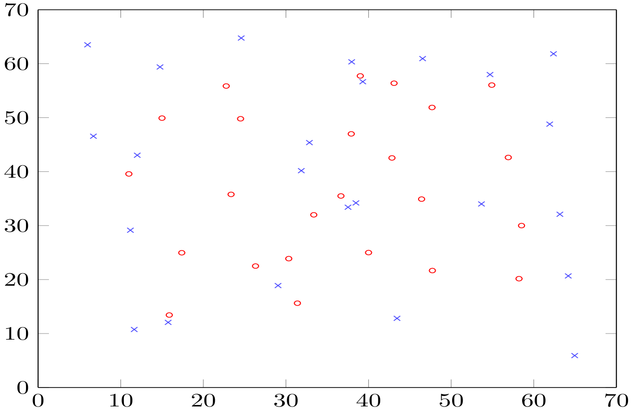
Total run time : 260 s

Locating 24 obstacles with noise-free data



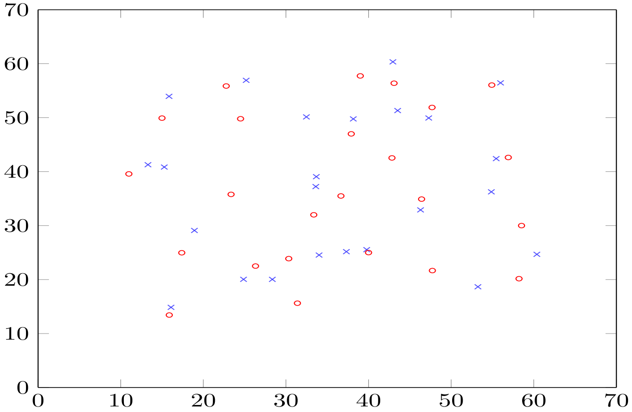
Total run time : 260 s

Locating 24 obstacles with noise-free data



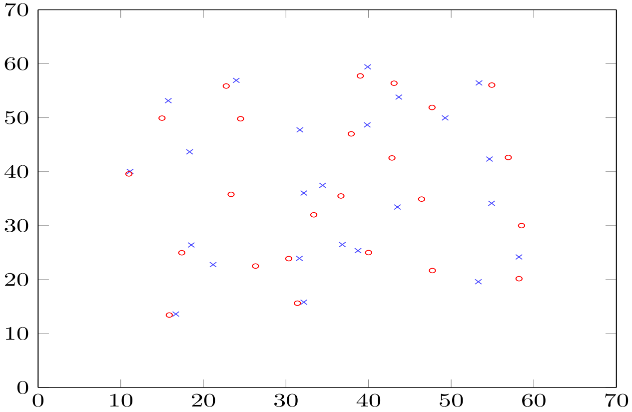
Total run time : 260 s

Locating 24 obstacles with noise-free data



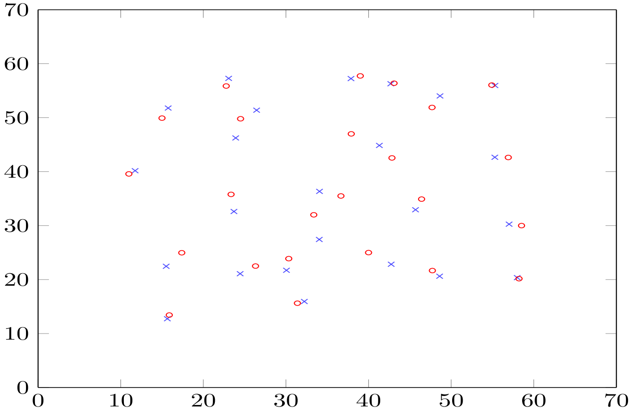
Total run time : 260 s

Locating 24 obstacles with noise-free data



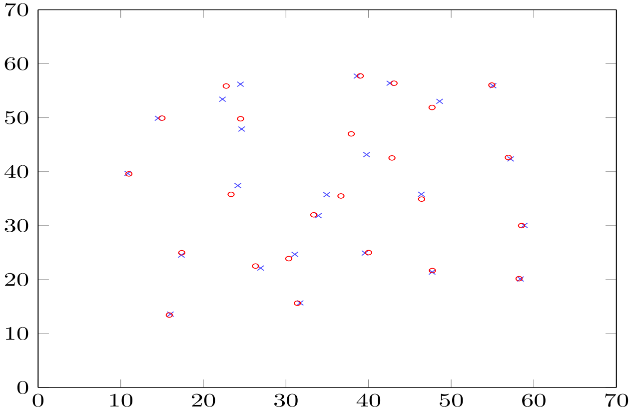
Total run time : 260 s

Locating 24 obstacles with noise-free data



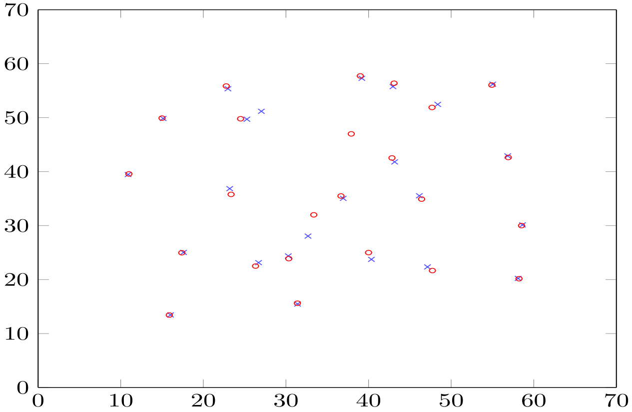
Total run time : 260 s

Locating 24 obstacles with noise-free data



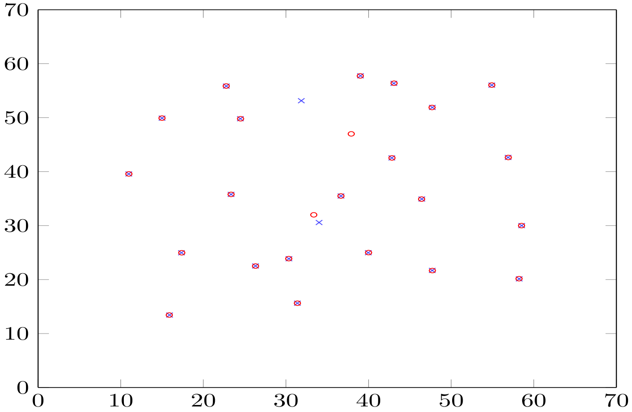
Total run time : 260 s

Locating 24 obstacles with noise-free data



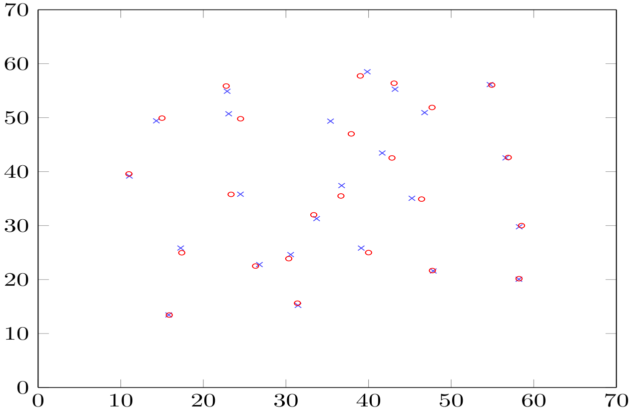
Total run time : 260 s

Locating 24 obstacles with noise-free data



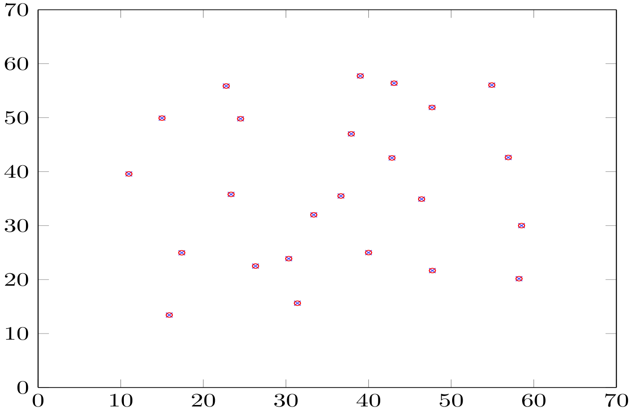
Total run time : 260 s

Locating 24 obstacles with noise-free data



Total run time : 260 s

Locating 24 obstacles with noise-free data



Total run time : 260 s

Two-stage reconstruction

Stage 1 : **Frequency marching** (from low to high)

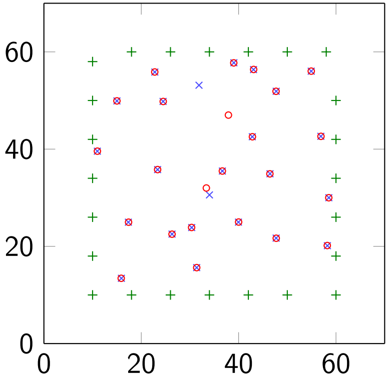
$$0.08 \;,\; 0.09 \;,\; 0.1:0.1:3$$

Two-stage reconstruction

Stage 1 : **Frequency marching** (from low to high)

$$0.08 \;,\; 0.09 \;,\; 0.1:0.1:3$$

$\rightsquigarrow$ most of the obstacles are accurately recovered, with the exception of two which are in the interior of the configuration.



Result of stage 1 (ite 2533):

$\mathcal{J} = 2.28$ (at $\kappa = 3.0$);

Rescaled position error = 12%,

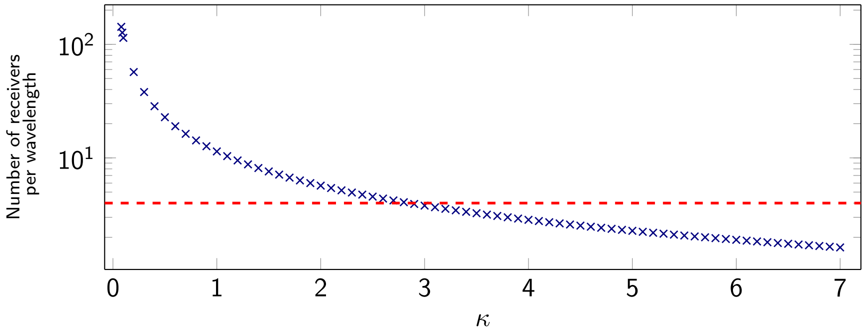
Run time = 200 s

$\circ$ = true positions ;

$+$ = initial guess ;

$\times$ = current reconstruction.

Problem of fixed number of data points at high frequencies



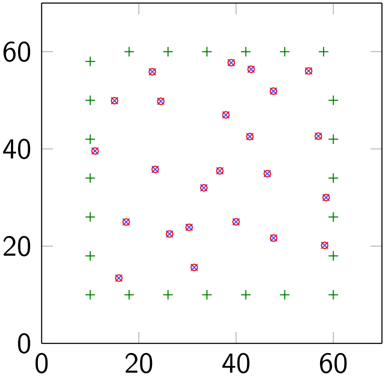
Receivers sampling per wavelength for varying wavenumber κ, corresponding to 128-point data set.

The red dashed line (- - -) corresponds with 4 points per wavelength (usually recommended to obtain enough information on the wave propagation)

Two-stage reconstruction (cnt)

Stage 2 - Recycle nine medium ranged frequencies:

0.1 , 0.2 , 0.3 , 0.5 , 1.0 , 1.5 , 2.0 , 2.5 , 3.0 .



Final reconstruction (2893 iterations)

$\mathcal{J} = 1.4 \times 10^{-7}$ (at $\kappa = 3.0$);

Scaled position error $= 2 \times 10^{-4}\%$.

Run time of stage 2 = 47s.

Total run time = 260 s.

$\circ$ = true positions ;

$+$ = initial guess ;

$\times$ = current reconstruction.

Plan

Conclusion

Apply full-waveform inversion with

- obstacle positions as retrieved parameters
- solver based on single-layer potential.
- limited number of radiation and with back-scattered data.
- fixed number of dataset for all frequencies.

- Quantitative reconstruction of obstacles positions using FWI
  - benefits from the fast solver,
  - promising results with 12 obstacles using noisy data,
  - several optimization methods implemented.
- Future: initial implementation of a qualitative method for more a priori information, then run FWI.

Thank you for your attention !