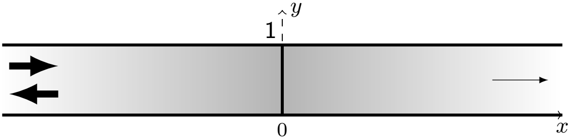
broadband enhanced transmission through symmetric diffusive slabs

5 - 9 nov. 2018

MecaWave

Élie CHÉRON
Simon FÉLIX, Vincent PAGNEUX
LAUM, CNRS UMR 6613, Le Mans Université

Enhanced transmission through an opaque barrier



Opaque barrier in an otherwise homogeneous quasi-1D waveguide.

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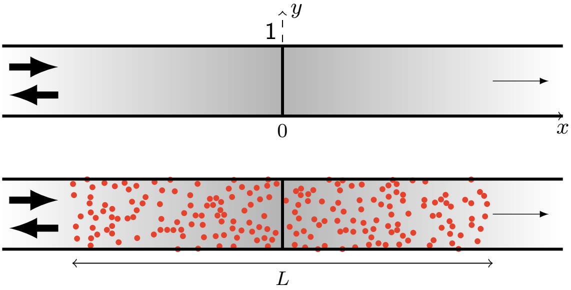

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Same as above, with randomly distributed scatterers on both sides of the barrier.

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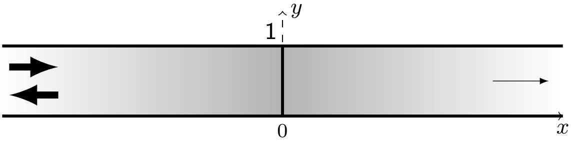

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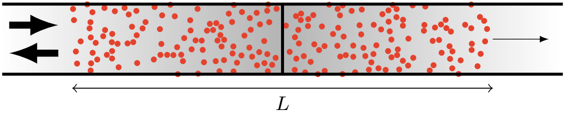

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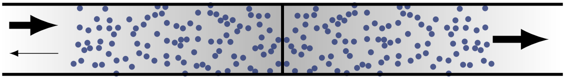

Same as above, but with a left-right symmetry of the scatterers position.

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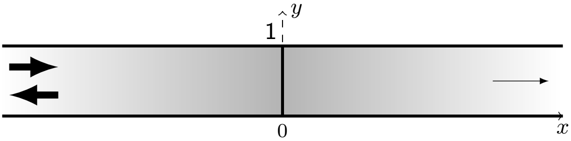

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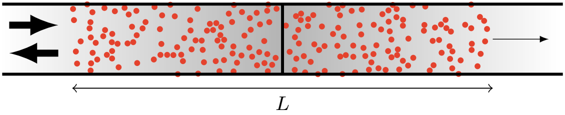

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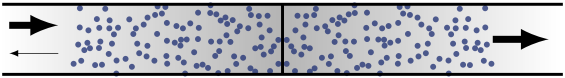

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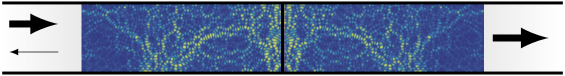

Typical open eigenchannel in the symmetrical case.

Multimodal scattering, conductance

transverse modes: $\phi_n'' = -n^2\pi^2\phi_n$

$$\psi(x,y) = \sum_n \left[a_n e^{ik_n(x+L/2)} + b_n e^{-ik_n(x+L/2)}\right]\phi_n(y)$$

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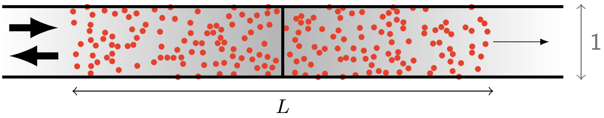

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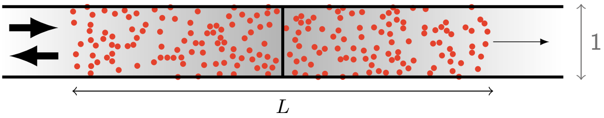

Transmission coefficients: $c_m = \mathrm{T}_{mn} a_n$

(T is a $N \times N$ matrix, N the number of propagating modes)

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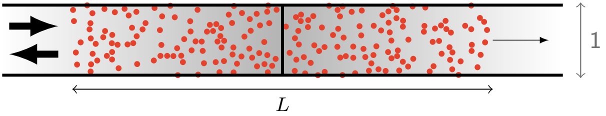


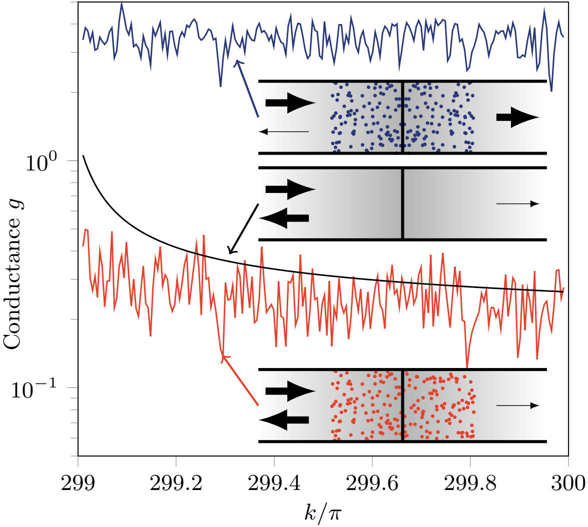
Transmission coefficients: $c_m = \mathsf{T}_{mn} a_n$

(T is a $N \times N$ matrix, N the number of propagating modes)

Conductance: $g = \mathrm{Tr}(\mathsf{T}\mathsf{T}^\dagger)$

(perfect transmission $\Leftrightarrow g = N$)

Broadband enhanced transmission



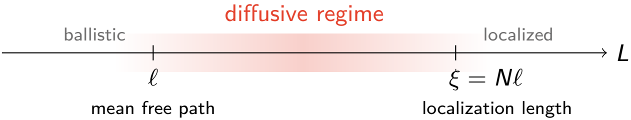
The opaque barrier, in the middle of a left-right symmetric disordered slab.

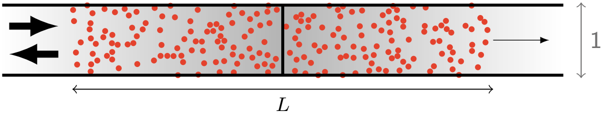
The opaque barrier alone:
$g = N\tau$, $\tau \ll 1$

The opaque barrier, in the middle of a disordered slab.

$N = 300$, $L = 5$, $\ell = 0.14$

Eigenchannels in the diffusive regime





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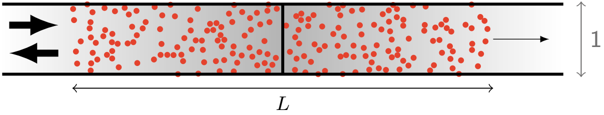

Conductance $g \lesssim N$ $g \propto L^{-1}$ $g \propto e^{-L/\xi}$

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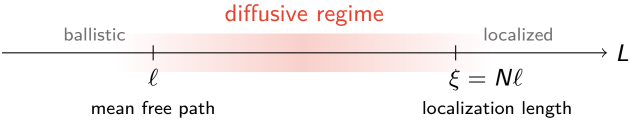
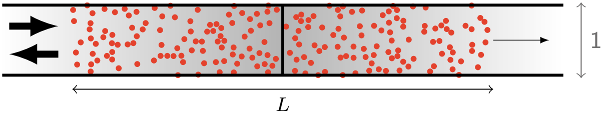
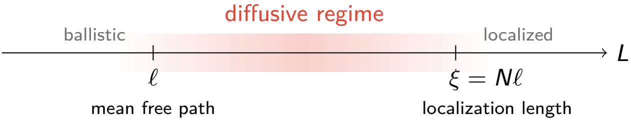


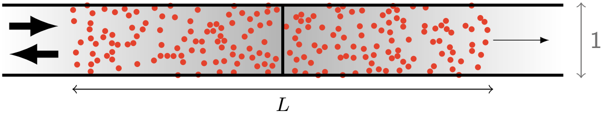


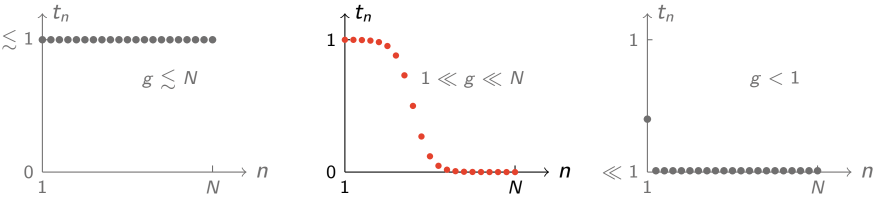
Conductance $\quad g \lesssim N \qquad g \propto L^{-1} \qquad g \propto e^{-L/\xi}$

Landauer $\longrightarrow$ $g = \mathrm{Tr}(TT^\dagger) = \sum_n t_n$ ← eigenvalues of $TT^\dagger$

Eigenchannels in the diffusive regime







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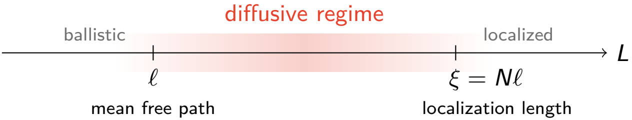
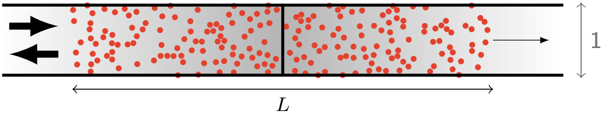
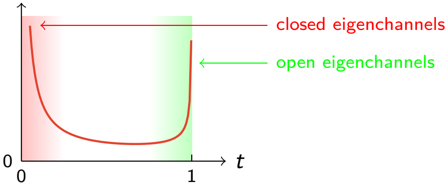




bimodal distribution of the eigenvalues:

$$P(t) = \frac{g}{2} \frac{1}{t\sqrt{1-t}}$$



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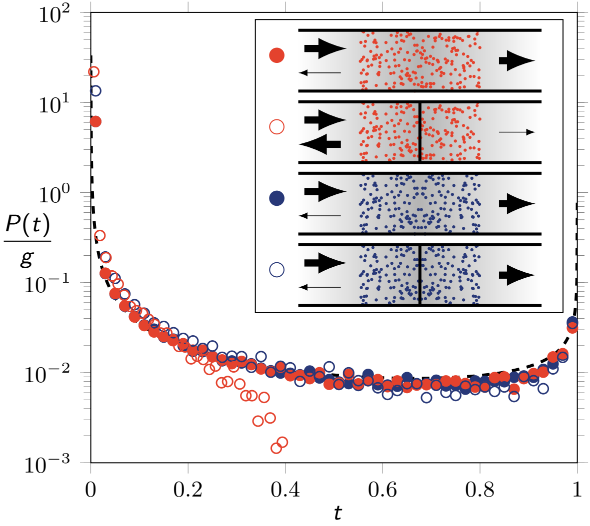
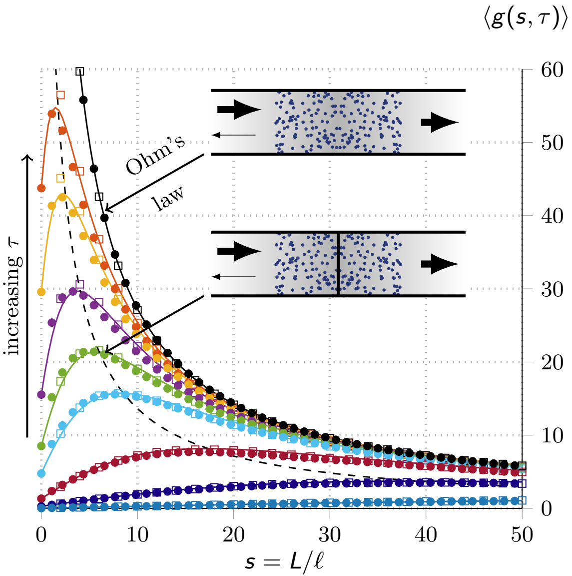


No symmetry.

Symmetric slabs.

$N = 300,\ L = 5,\ \ell = 0.14$

Optimized conductance enhancement



Ohm's law:
$\langle g(s,1)\rangle \simeq g_{\rm D}(s) = \dfrac{N}{1+s}$

Colored dots: full wave numerics, averaged over 100 realizations of the disorder.

Squares: Random Matrix Theory.

Scaling model

inspired by Whitney *et al.*, PRL **102**,186802 (2009).

$$\langle g(s,\tau)\rangle = \frac{\zeta_1(s)N\tau}{1+\zeta_2(s)N\tau}$$

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$$\langle g(s,\tau)\rangle \simeq \frac{N\tau}{(1+s)\tau + \dfrac{1-\tau}{1+\eta s}}$$

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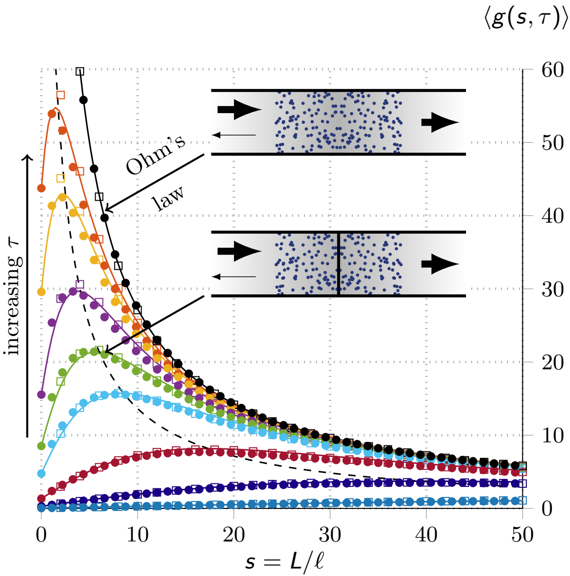

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Maximum conductance enhancement reached when $s = s_{\text{opt}} = \sqrt{\dfrac{\tau+1}{\eta\tau}} - \dfrac{1}{\eta}$

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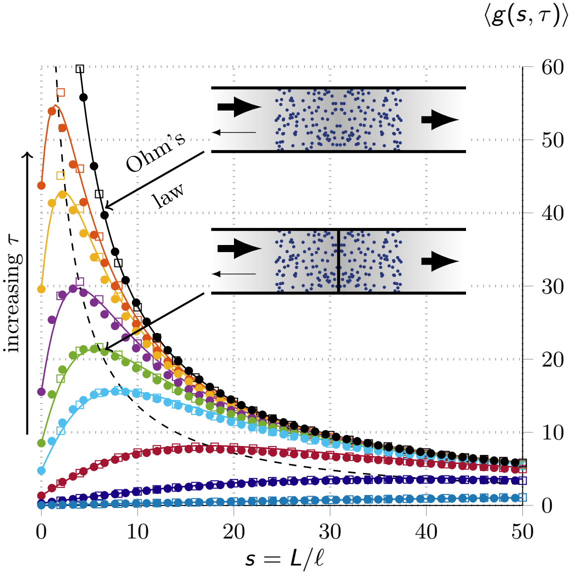
Optimized conductance enhancement



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Solid lines: scaling model.

Dashed line: $\dfrac{g_D}{2}$

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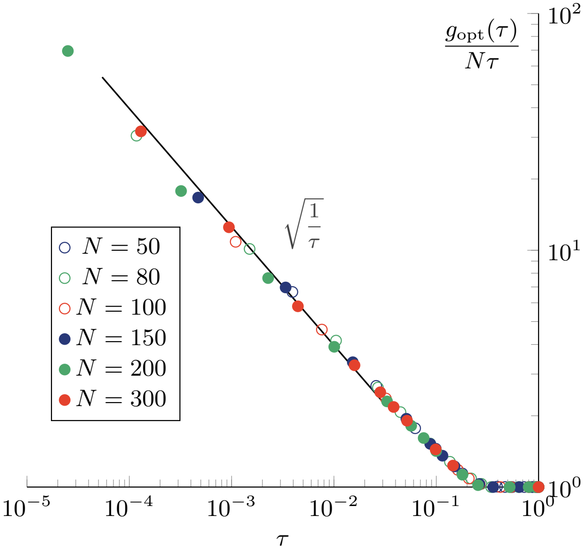
Then, for an opaque barrier, $g_{\text{opt}}(\tau \ll 1) \simeq \dfrac{g_{\text{D}}(s_{\text{opt}})}{2}$

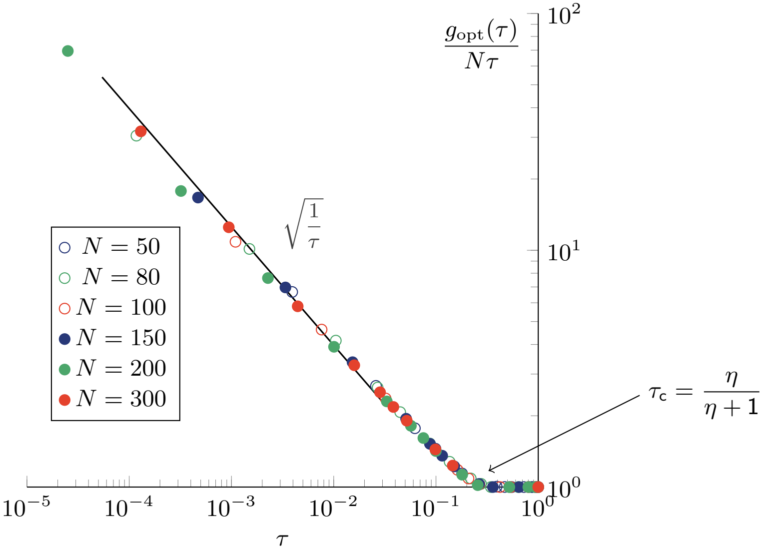
The maximum conductance enhancement scales as: $\dfrac{g_{\text{opt}}(\tau \ll 1)}{N\tau} \simeq \dfrac{1}{2}\sqrt{\dfrac{\eta}{\tau}}$

Scaling



$\frac{g_{\text{opt}}(\tau)}{N\tau}$
10^2
10^1
10^0
$\sqrt{\frac{1}{\tau}}$
$N = 50$
$N = 80$
$N = 100$
$N = 150$
$N = 200$
$N = 300$
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0
τ

Scaling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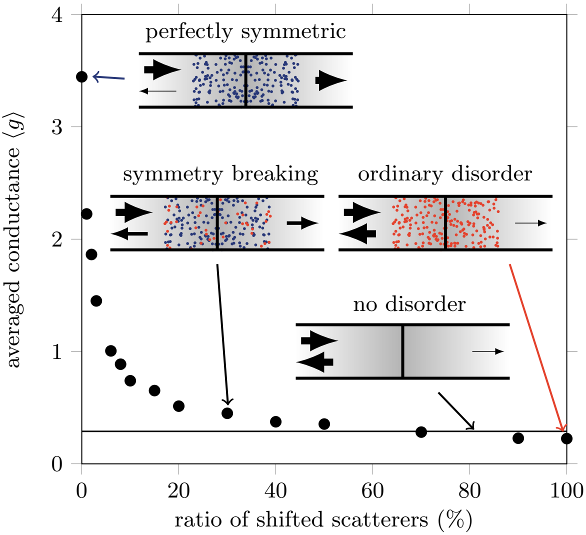
$\tau_c = \frac{\eta}{\eta + 1}$
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
τ

Symmetry breaking



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