# Onde acoustique avec changement de phase induit


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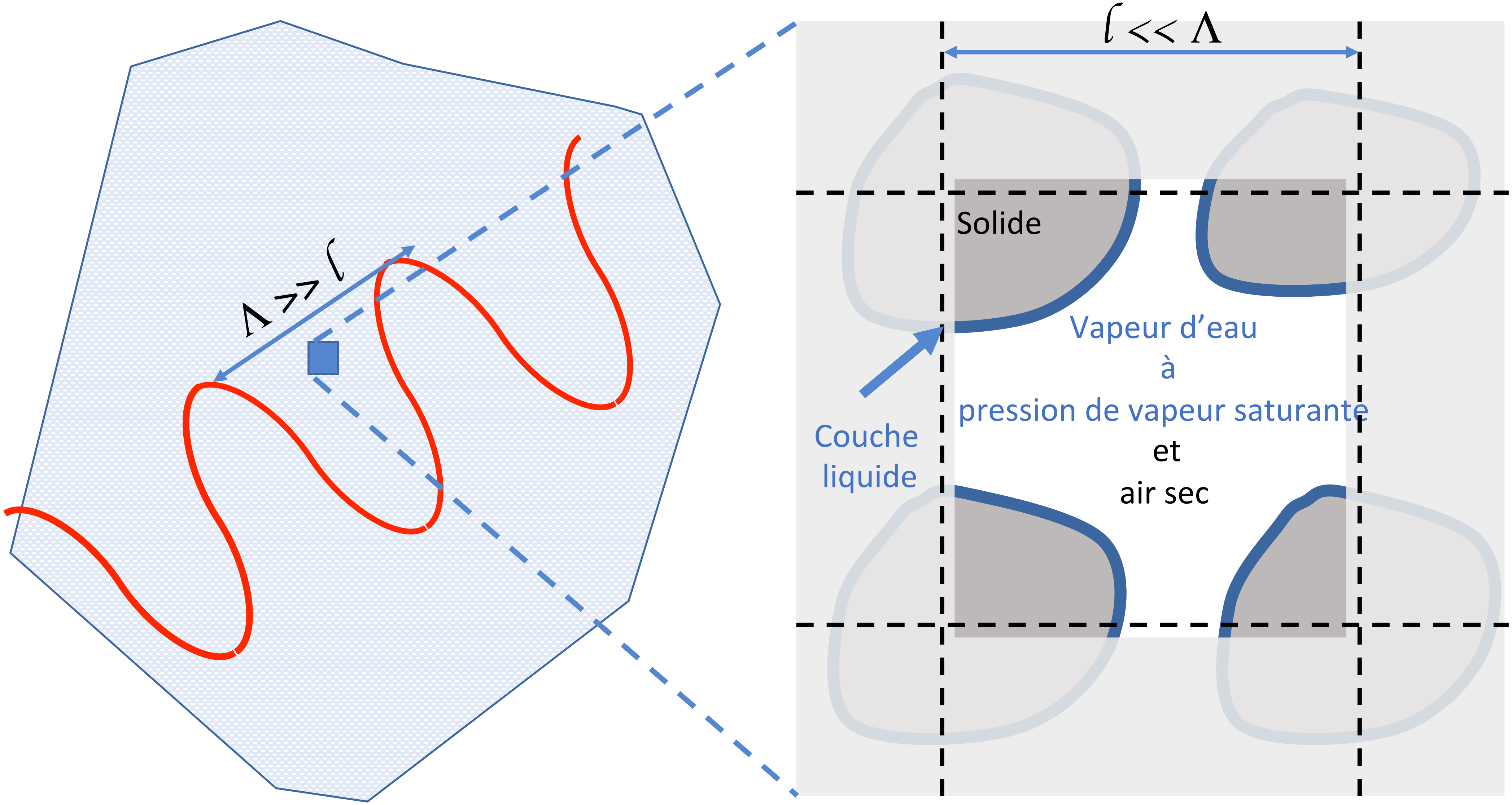
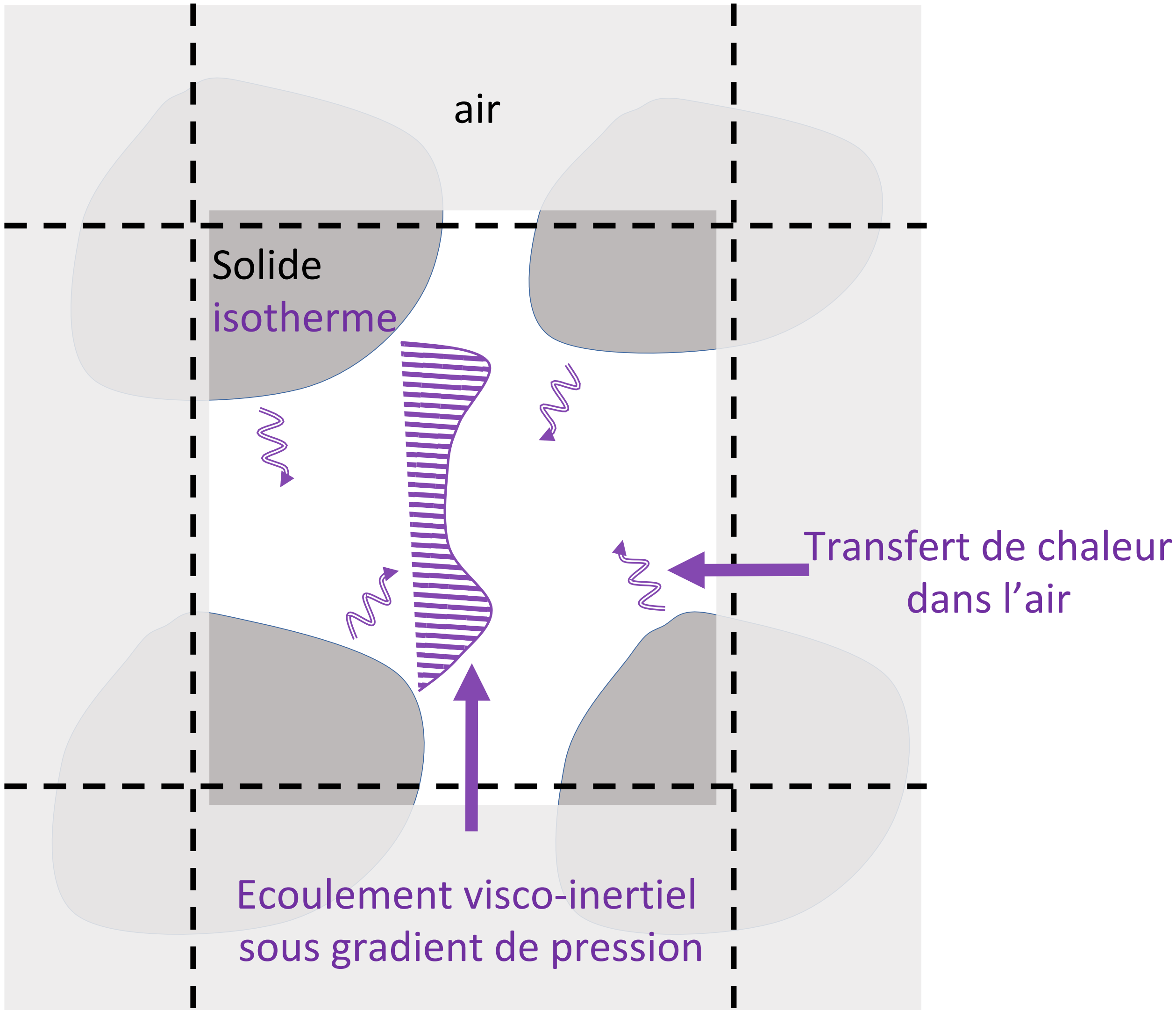
3ème colloque du GdR MecaWave
8-12 mai 2023
Centre IGESA, 83400 Porquerolles

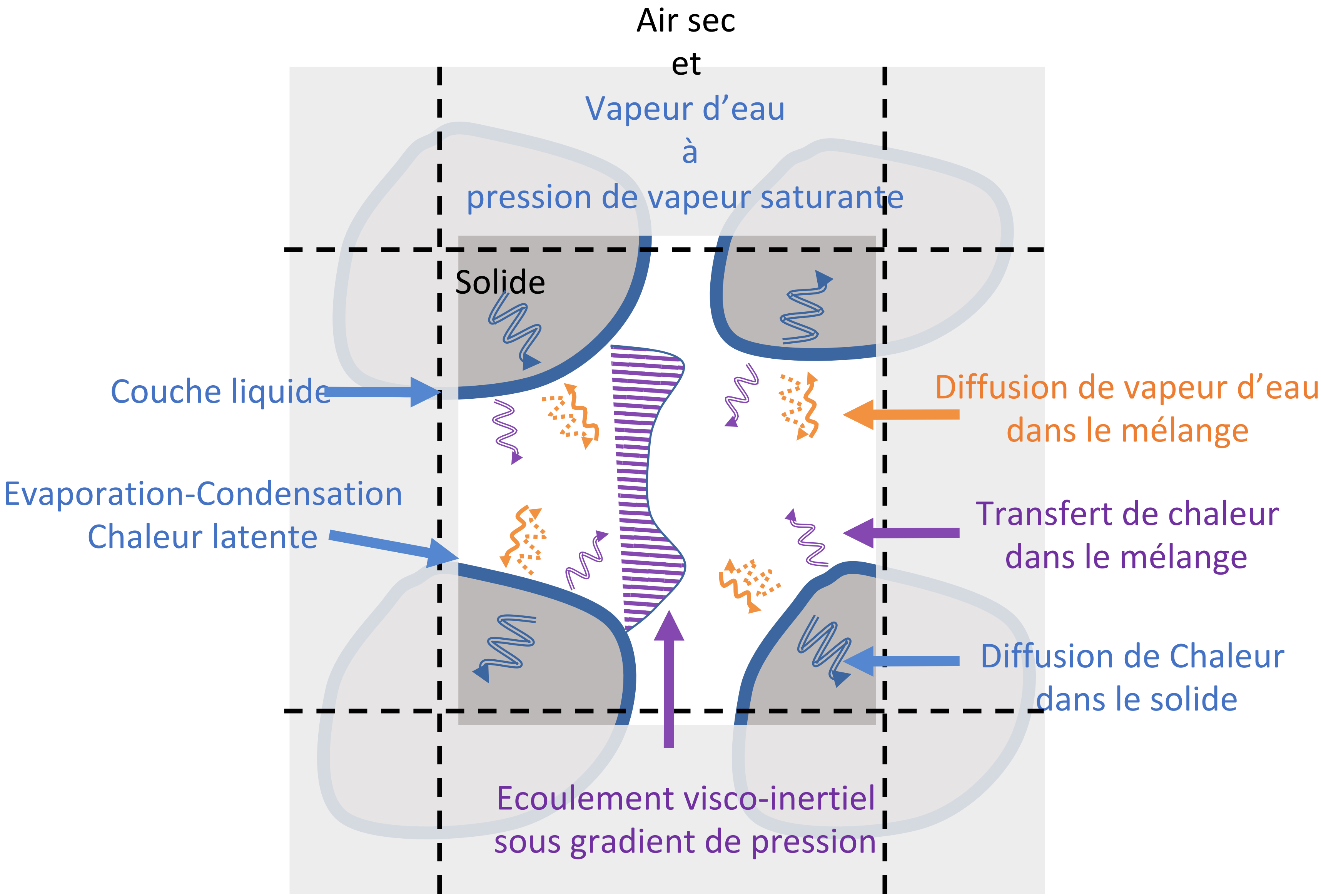
# Acoustique en Milieu poreux humide



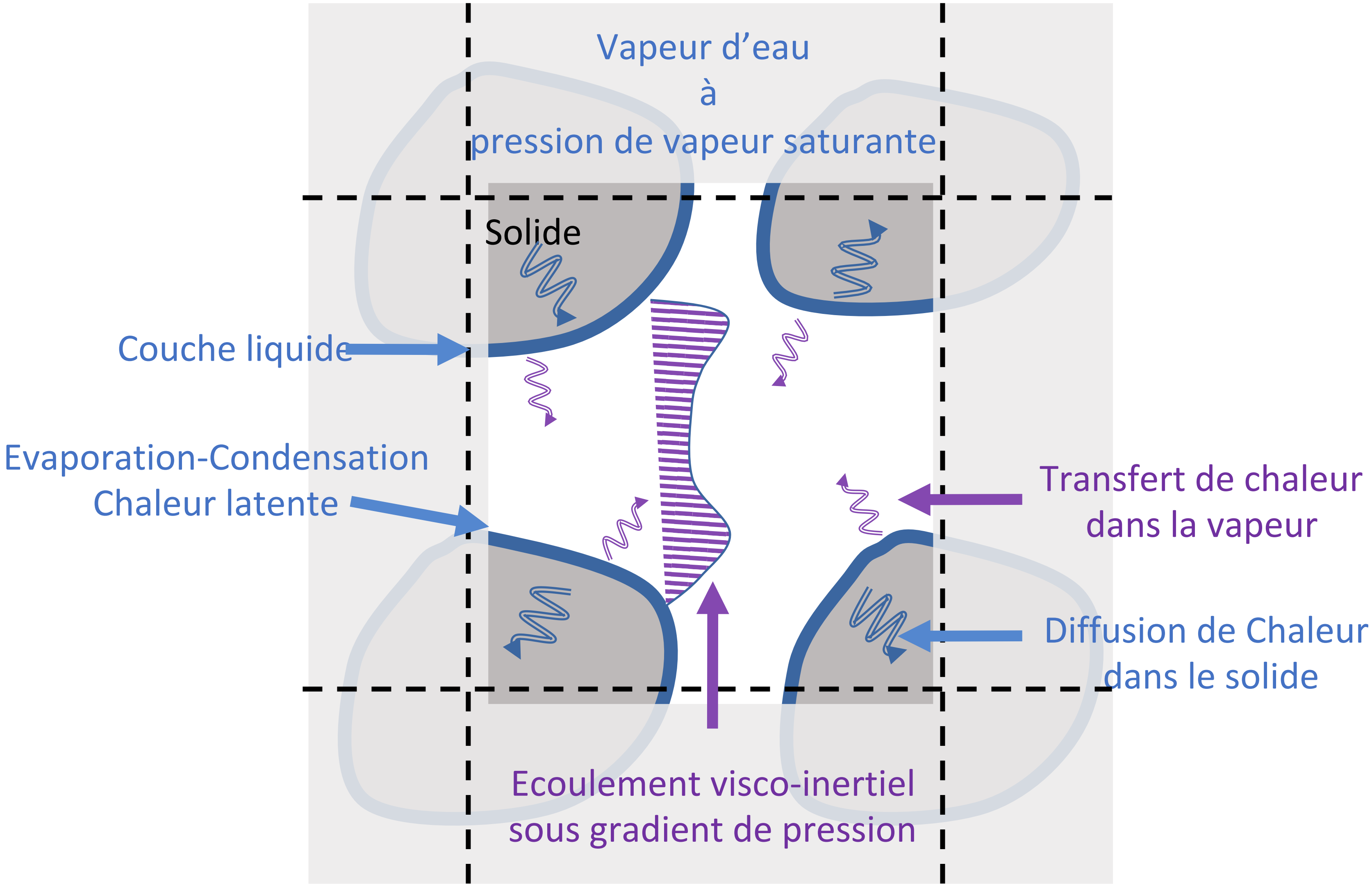
# Physique des ondes en milieu sec



# Physique  en milieu humide



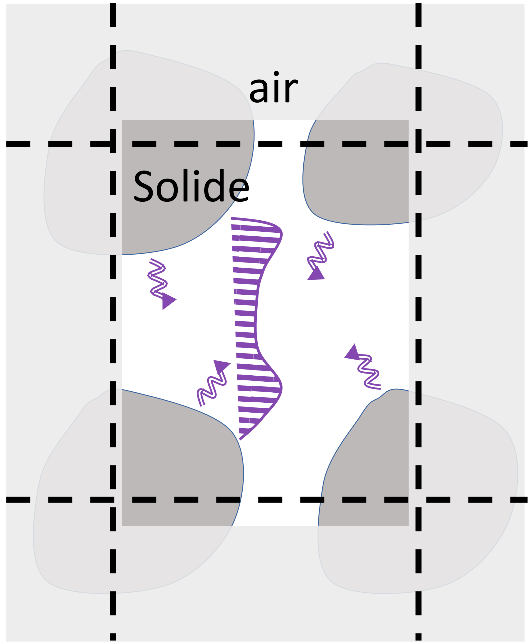
# Milieu saturé en vapeur d’eau



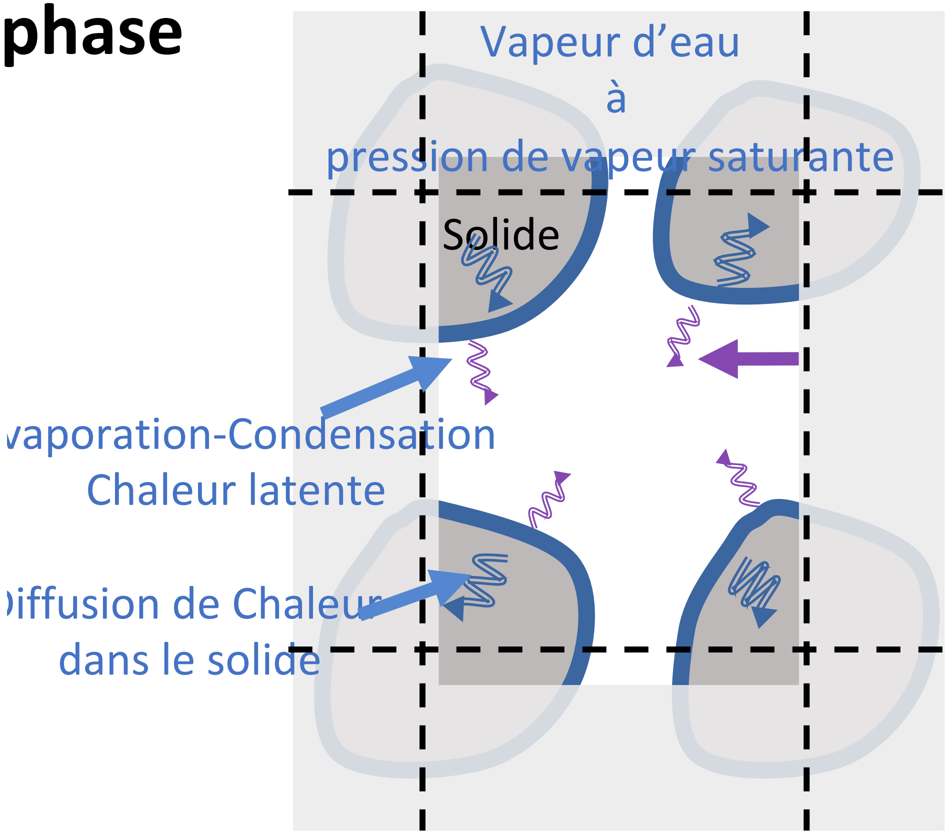
# Le systéme d'équations classique

**Perturbation en régime harmonique e$^{+i\omega t}$**

$$
\begin{aligned}
\mathbf{div}(\boldsymbol{\sigma}) &= i\omega\rho^e\mathbf{v} \\
\boldsymbol{\sigma} &= 2\eta\mathbf{D}(\mathbf{v}) - p\mathbf{I} \\
\rho^e\mathrm{div}(\mathbf{v}) + i\omega\rho &= 0 \\
-\mathrm{div}(\kappa\mathbf{grad}\tau) + i\omega\rho^e c_p\tau &= i\omega p \\
\frac{p}{P^e} &= \frac{\rho}{\rho^e} + \frac{\tau}{T^e}
\end{aligned}
$$



# Les équations de changement de phase




## Clapeyron et chaleur latente $\mathcal{L}$

**!** $$\tau = \tau_w = \tau_\Gamma = p_v \frac{T^e}{\mathcal{L}\rho_v^e}$$

**!** $$(-\kappa_w \mathbf{grad}\tau_w + \kappa \mathbf{grad}\tau).\mathbf{n} = -\mathcal{L}\mathbf{j}.\mathbf{n}$$

$$-\mathrm{div}(\kappa_{ws}\mathbf{grad}\tau_{ws}) + i\omega(\rho c)_{ws}\tau_{ws} = 0$$

## Vitesse à l'interface $\Gamma$

$$\mathbf{v}_w = \mathbf{0}$$

$$(\mathbf{v} - \mathbf{v}_w).\mathbf{n} = \left(\frac{1}{\rho_v^e} - \frac{1}{\rho_w}\right)\mathbf{j}.\mathbf{n}$$

**!** $$\mathbf{v}.\mathbf{n} = \frac{1}{\rho_v^e}\mathbf{j}.\mathbf{n}$$

## Analyse physique $\varepsilon = \dfrac{l}{L} << 1$

**Acoustique à grande longeur d'onde : L = $\Lambda/2\pi$**

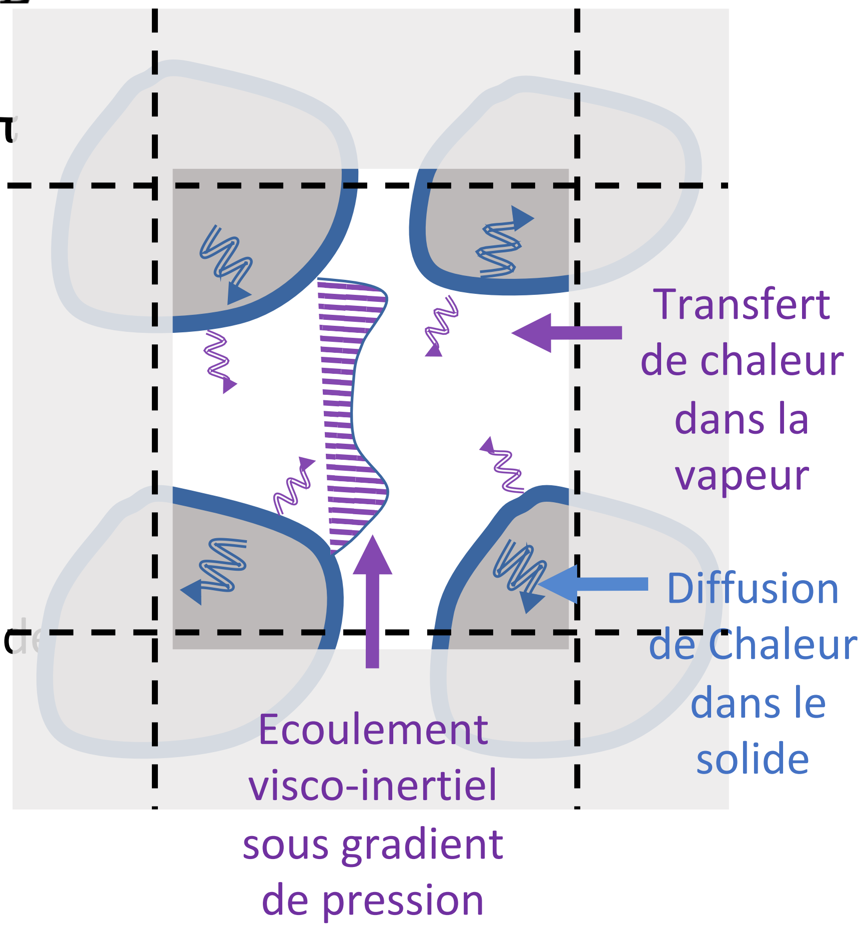
$$\mathbf{grad}(p) = O(\frac{p}{L}) \quad ; \quad \mathrm{div}(\mathbf{v}) = O(\frac{v}{L})$$

**Quantités oscillant à l'échelle des pores : $\ell$ << $\Lambda/2\pi$**

Temperature
Vitesse
Masse volumique

**Contrastes**

Conductivité thermique vapeur << eau/solide
Masse volumique vapeur << eau
Vitesse sur $\Gamma$ << Vitesse dans $\Omega_g$



## Homogénéisation

Procédure usuelle par développement asymptotique

en

puissance de $\varepsilon$

des

variables exprimées en double échelle $x$, $y = \varepsilon^{-1}x$

# Système renormalisé en double échelles x, y = $\varepsilon^{-1}$x

$$\begin{aligned}
&\text{in} \quad \Omega_g \\
-\varepsilon^2 \mathrm{div}(\kappa \,\mathbf{grad}\tau) + i\omega\rho^e c_p \tau &= i\omega p \\
\rho^e \mathrm{div}(\mathbf{v}) + i\omega\rho &= 0 \\
-\varepsilon^2 \mathrm{div}(\kappa \,\mathbf{grad}\tau) + i\omega\rho^e c_p \tau &= i\omega p \\
\frac{p}{P^e} &= \frac{\rho}{\rho^e} + \frac{\tau}{T^e} \\
&\text{in} \quad \Omega_{ws} \\
-\varepsilon^2 \mathrm{div}(\kappa_{ws} \mathbf{grad}\tau_{ws}) + i\omega(\rho c)_{ws}\tau_{ws} &= 0 \\
&\text{on} \quad \Gamma \\
(-\varepsilon\kappa_w \mathbf{grad}\tau_w + \varepsilon^2 \kappa \,\mathbf{grad}\tau).\mathbf{n} &= - \mathcal{L}\mathbf{j}.\mathbf{n} \\
\mathbf{v}.\mathbf{n} = \varepsilon \frac{1}{\rho_v^e}\mathbf{j}.\mathbf{n}
\end{aligned}$$

$$\triangle_x(q) = O\left(\frac{q}{L^2}\right) = O\left(\frac{q}{L^2}\right)\frac{\ell^2}{L^2} = O\left(\frac{q}{\ell^2}\right)\varepsilon^2$$

$$\kappa \,\mathbf{grad}\tau.\mathbf{n} \ll -\kappa_w \mathbf{grad}\tau_w \approx \mathcal{L}\mathbf{j}.\mathbf{n}$$

$$\begin{aligned}
\mathrm{d}(S\rho^e|v|)/S\rho^e|v| &\approx \ell/L = \varepsilon \\
\mathrm{d}(S\rho^e|v|) &\approx |j|\Gamma \\
|v|_\Gamma/|v| &\approx |j|\Gamma/S\rho^e|v| \approx l/L = \varepsilon
\end{aligned}$$

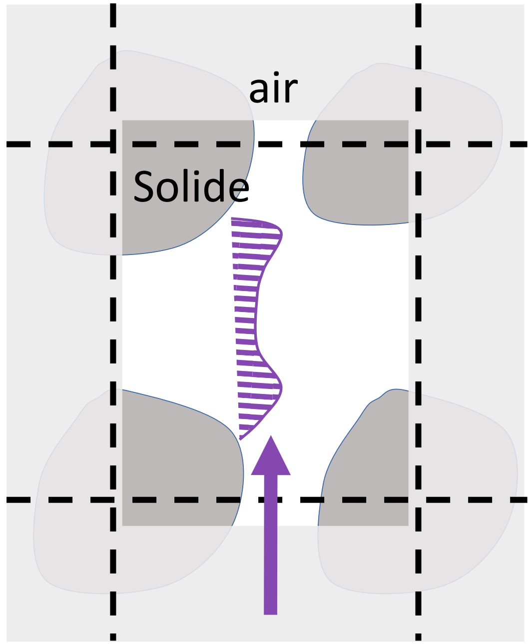
# Loi de Darcy dynamique

**Ordre dominant**

$$\mathbf{grad}_y p^{(0)} = 0 \qquad ; \qquad p^{(0)} = P(\mathbf{x})$$

**Ordre suivant**

$$\begin{aligned} -\mathbf{grad}_y p^{(1)} - \mathbf{grad}_x P + \eta \Delta_y \mathbf{v}^{(0)} &= i\omega\rho^e \mathbf{v}^{(0)} \\ \mathrm{div}_y(\mathbf{v}^{(0)}) &= 0 \\ \boxed{\mathbf{v}^{(0)} = \mathbf{0}} & \\ p^{(1)}; \mathbf{v}^{(0)} \quad &: \quad \Omega - \text{periodic} \end{aligned}$$




Ecoulement visco-inertiel sous gradient de pression

**Résolution classique**

$$\mathbf{v}^{(0)}(\mathbf{x}, \mathbf{y}) = -\mathbf{k}(\omega, \mathbf{y})\mathbf{grad}_x P \qquad \text{soit} \qquad \boxed{V(\mathbf{x}) = \langle \mathbf{v}^{(0)} \rangle = -\mathbf{K}(\omega)\mathbf{grad}_x P}$$

**Le changement de phase n'affecte pas la loi de Darcy dynamique**

# Température d'interface eau-vapeur

**Equilibre liquide vapeur : Relation de Clapeyron**

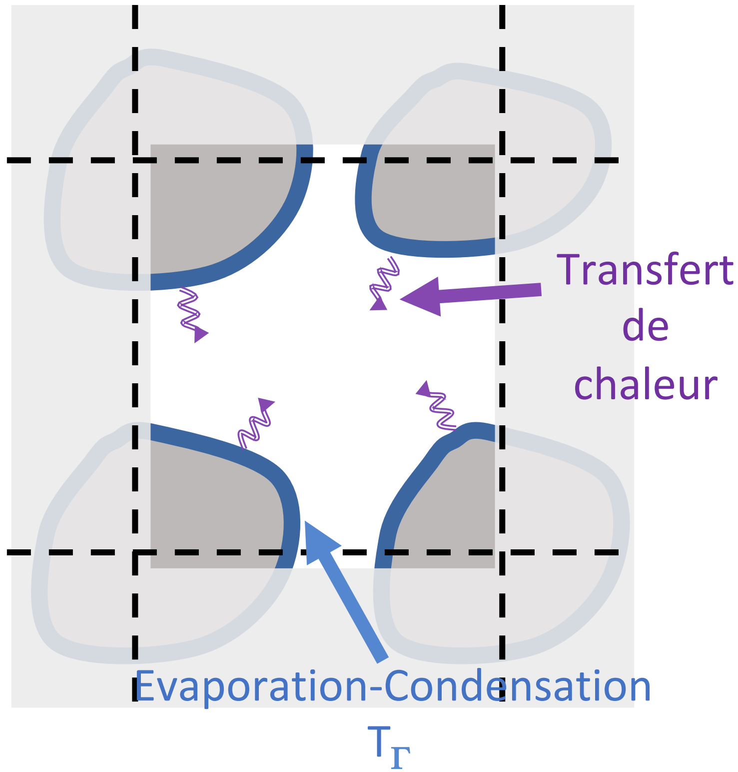
$$\tau^{(0)} = \tau_w^{(0)} = T_\Gamma(\mathbf{x}) = P(\mathbf{x})\frac{T^e}{\mathcal{L}\rho_v^e} \quad \text{on} \quad \Gamma$$

❗ **Uniforme ≠ 0 sur $\Gamma$**

# Vapeur - Transfert thermique

**Ordre dominant**

$$\begin{aligned}
-\mathrm{div}_y(\kappa \mathbf{grad}_y \tau^{(0)}) + i\omega\rho^e c_p \tau^{(0)} &= i\omega P \quad \text{in} \quad \Omega_g \\
\boxed{\tau^{(0)} = T_\Gamma} & \quad \text{on} \quad \Gamma \\
\tau^{(0)} &: \quad \Omega - \text{periodic}
\end{aligned}$$




**Résolution classiqu** $\widetilde{\tau} = \tau^{(0)} - T_\Gamma$

$$\begin{aligned}
-\mathrm{div}_y(\kappa \mathbf{grad}_y \widetilde{\tau}) + i\omega\rho^e c_p \widetilde{\tau} &= i\omega(P - \rho^e c_p T_\Gamma) \quad \text{in} \quad \Omega_g \\
\widetilde{\tau} &= 0 \quad \text{on} \quad \Gamma \\
\widetilde{\tau} &: \quad \Omega - \text{periodic}
\end{aligned}$$

**Solution particulière complexe**

$$\begin{aligned}
-\mathrm{div}_y(\mathbf{grad}_y \theta) - \frac{i\omega\rho^e c_p}{\kappa}\theta &= 1 \quad \text{in} \quad \Omega_g \\
\theta &= 0 \quad \text{on} \quad \Gamma \\
\theta &: \quad \Omega - \text{periodic}
\end{aligned}$$

# Vapeur - Température et masse volumique

**Température**

$$\tau^{(0)} = T_\Gamma + (\frac{P}{\rho^e c_p} - T_\Gamma)\frac{i\omega\rho^e c_p \theta(\mathbf{y})}{\kappa}$$

**Masse volumique**

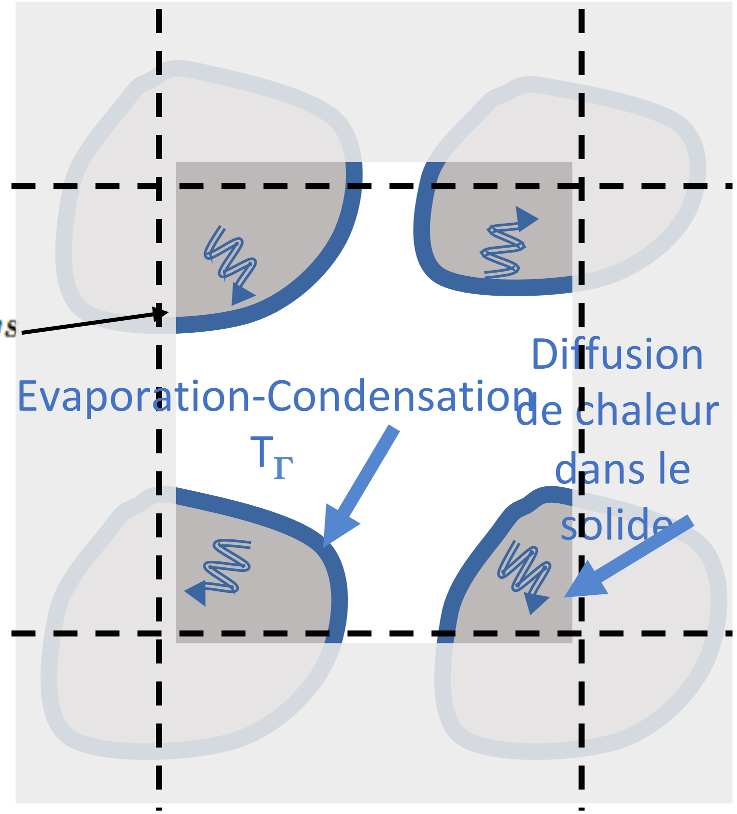
$$\frac{\rho^{(0)}}{\rho^e} = \frac{P}{P^e} - \frac{\tau^{(0)}}{T^e} = \frac{P}{P^e} - \frac{T_\Gamma}{T^e} - (\frac{P}{\rho^e c_p T^e} - \frac{T_\Gamma}{T^e})\frac{i\omega\rho^e c_p \theta(\mathbf{y})}{\kappa}$$

$$\boxed{\langle\frac{\rho^{(0)}}{\rho^e}\rangle = \frac{P}{P^e} - \frac{T_\Gamma}{T^e} - (\frac{P}{\rho^e c_p T^e} - \frac{T_\Gamma}{T^e})\frac{i\omega\rho^e c_p \langle\theta\rangle}{\kappa}}$$

# Système eau & solide - Transfert thermique

**Ordre dominant**

$$
\begin{aligned}
-\mathrm{div}_y(\kappa_{ws}\mathbf{grad}_y\tau_{ws}^{(0)}) + i\omega(\rho c)_{ws}\tau_{ws}^{(0)} &= 0 \quad \text{in} \quad \Omega_{ws} \\
\boxed{\tau_{ws}^{(0)} = T_\Gamma} & \quad \text{on} \quad \Gamma \\
\kappa_{ws}\mathbf{grad}_y\tau_{ws}^{(0)}.n \quad \text{and} \quad \tau_{ws}^{(0)} &: \text{continuous on} \quad \Gamma_{ws} \\
\tau_{ws}^{(0)} &: \Omega - \text{periodic}
\end{aligned}
$$




**Résolution**

$$
\tau_{ws}^{(0)} = T_\Gamma\left(1 - i\omega\langle\rho c\rangle_{ws}\frac{\Theta(\mathbf{y})}{\kappa_w}\right)
$$

**Solution particulière complexe $\Omega$ −periodic**

$$
\begin{aligned}
-\mathrm{div}_y\left(\frac{\kappa_{ws}}{\kappa_w}\mathbf{grad}_y\Theta\right) + \frac{i\omega(\rho c)_{ws}}{\kappa_w}\Theta &= \frac{(\rho c)_{ws}}{\langle\rho c\rangle_{ws}} \quad \text{in} \quad \Omega_{ws}, \\
\Theta &= 0 \quad \text{on} \quad \Gamma \\
\kappa_{ws}\mathbf{grad}_y\Theta.n \quad \text{and} \quad \Theta &: \text{continuous on} \quad \Gamma_{ws}
\end{aligned}
$$

# Flux de masse changeant de phase

**Température**

$$\tau_{ws}^{(0)} = T_\Gamma(1 - i\omega\langle\rho c\rangle_{ws}\frac{\Theta(\mathbf{y})}{\kappa_w})$$

**Flux local de chaleur**

$$\text{-}\mathcal{L}\mathbf{j}^{(0)}.\mathbf{n} = -\kappa_w \mathbf{grad}_y \tau_w^{(0)}.\mathbf{n}$$

**Masse changeant de phase dans Ω**

$$\boxed{\begin{aligned} \int_\Gamma \mathbf{j}^{(0)}.\mathrm{nds} &= \frac{1}{\mathcal{L}}\int_\Gamma \kappa_w \mathbf{grad}_y \tau_w^{(0)}.\mathrm{n}ds \\ &= \frac{1}{\mathcal{L}}\int_{\Omega_{ws}} i\omega(\rho c)_{ws}\tau_{ws}^{(0)}\mathrm{d}v = \frac{T_\Gamma}{\mathcal{L}} i\omega\langle\rho c\rangle_{ws}(1 - \frac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w})\Omega_{ws} \end{aligned}}$$

# Conservation de la masse

**Ordre dominant : incompressibilité** $\mathrm{div}_y(\mathbf{v}^{(0)}) \;=\; 0$

**Premier ordr**

$$\mathbf{v}^{(1)}.\mathbf{n} \;=\; \frac{1}{\rho^e}\mathbf{j}^{(0)}.\mathbf{n} \quad \text{on} \quad \Gamma$$

$$\mathrm{div}_y(\mathbf{v}^{(1)}) + \mathrm{div}_x(\mathbf{v}^{(0)})) + i\omega\frac{\rho^{(0)}}{\rho^e} \;=\; 0 \quad \text{in} \quad \Omega_g$$

**Par intégration**

$$\int_{\Omega_g} \mathrm{div}_y(\mathbf{v}^{(1)})\mathrm{dv} + \int_{\Omega_g} \mathrm{div}_x(\mathbf{v}^{(0)}))\mathrm{dv} + \int_{\Omega_g} i\omega\frac{\rho^{(0)}}{\rho^e}\mathrm{dv} \;=\; 0$$

$$\boxed{\frac{1}{\Omega_g}\int_\Gamma \frac{1}{\rho^e}\mathbf{j}^{(0)}.\mathbf{n}\mathrm{ds} + \mathrm{div}_x(\mathbf{V}) + i\omega\langle\frac{\rho^{(0)}}{\rho^e}\rangle \;=\; 0}$$

# Compressibilité effective

**Rappel expressions de $T_\Gamma$, $\rho$ et j**

$$
\begin{aligned}
\frac{T_\Gamma}{T^e} &= \frac{P}{P^e}\frac{P^e}{\mathcal{L}\rho_v^e} \\
\langle\frac{\rho^{(0)}}{\rho^e}\rangle &= \frac{P}{P^e} - \frac{T_\Gamma}{T^e} - \left(\frac{P}{\rho^e c_p T^e} - \frac{T_\Gamma}{T^e}\right)\frac{i\omega\rho^e c_p\langle\theta\rangle}{\kappa} \\
\frac{1}{\Omega_g}\int_\Gamma \frac{1}{\rho_v^e}\mathbf{j}^{(0)}.\mathbf{n}ds &= i\omega\frac{T_\Gamma}{\mathcal{L}\rho_v^e}\langle\rho c\rangle_{ws}\left(1 - \frac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w}\right)\frac{\Omega_{ws}}{\Omega_g}
\end{aligned}
$$

**Effet de changement de phase**

$$
\begin{aligned}
\mathrm{div}_x(\mathbf{V}) &= -i\omega\frac{P}{P^e}\left(1 - \boxed{\frac{P^e}{\mathcal{L}\rho_v^e}}\left(1 + \left(\boxed{\frac{\mathcal{L}\rho_v^e}{\rho^e c_p T^e}} - 1\right)\frac{i\omega\rho^e c_p\langle\theta\rangle}{\kappa}\right)\right) \\
&\quad - i\omega\frac{P}{P^e}\left(\boxed{\frac{P^e T^e\langle\rho c\rangle_{ws}}{(\mathcal{L}\rho_v^e)^2}}\left(1 - \frac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w}\right)\frac{\Omega_{ws}}{\Omega_g}\right)
\end{aligned}
$$

# Quantification

$$\mathrm{div}_x(\mathbf{V}) = -i\omega\frac{P}{P^e}\left(1-\boxed{\frac{P^e}{\mathcal{L}\rho_v^e}}\left(1+\left(\boxed{\frac{\mathcal{L}\rho_v^e}{\rho^e c_p T^e}}-1\right)\frac{i\omega\rho^e c_p\langle\theta\rangle}{\kappa}\right)\right)$$
$$-i\omega\frac{P}{P^e}\left(\boxed{\frac{P^eT^e\langle\rho c\rangle_{ws}}{(\mathcal{L}\rho_v^e)^2}}\left(1-\frac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w}\right)\frac{\Omega_{ws}}{\Omega_g}\right)$$

$\mathcal{L}$ = 22.5x $10^5$ J/kg

$\gamma$ = 1.4

$$\frac{P^eT^e\langle\rho c\rangle_{ws}}{(\mathcal{L}\rho_v^e)^2} = \left(\frac{P^e}{\mathcal{L}\rho_v^e}\right)^2\frac{\langle\rho c\rangle_{ws}}{\rho^e c_p}\frac{\gamma}{\gamma-1} \approx \boxed{8}$$
$$\frac{P^e}{\mathcal{L}\rho_v^e} = \boxed{3.710^{-2}}$$
$$\frac{\mathcal{L}\rho_v^e}{\rho^e c_p T^e} = \frac{\mathcal{L}\rho_v^e}{P^e}\frac{\gamma-1}{\gamma} = \boxed{7.7}$$

| **Parametres** | Liquid | Vapor | Solid |
|---|---|---|---|
| Density $\rho^e$ [kg/m$^3$] | $10^3$ | 1.2 | $2.5\times10^3$ |
| Bulk modulus $K$ [Pa] | $2\ 10^9$ | $1.4\times10^5$ | $40\times10^9$ |
| Dynamic viscosity $\mu$ [Pa.s] | $10^{-3}$ | $20\times10^{-6}$ | |
| Kinematic viscosity $\nu$ [m$^2$/s] | $10^{-6}$ | $15\times10^{-6}$ | |
| Conductivity $k$ [W/K.m] | 0.6 | 0.026 | 3 |
| Heat capacity $C_p$ [J/K.kg] | $4.18\times10^3$ | $10^3$ | $0.8\times10^3$ |
| Diffusivity $d$ [m$^2$/s] | $1.4\times10^{-7}$ | $2.1\times10^{-5}$ | $1.5\times10^{-6}$ |

## Dépendance fréquentielle

$$\operatorname{div}_x(\mathbf{V}) = -i\omega\frac{P}{P^e}\left(1-\frac{P^e}{\mathcal{L}\rho_v^e}\left(1+\left(\frac{\mathcal{L}\rho_v^e}{\rho^e c_p T^e}-1\right)\boxed{\frac{i\omega\rho^e c_p\langle\theta\rangle}{\kappa}}\right)\right)$$
$$-i\omega\frac{P}{P^e}\left(\frac{P^e T^e\langle\rho c\rangle_{ws}}{(\mathcal{L}\rho_v^e)^2}\left(1-\boxed{\frac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w}}\right)\frac{\Omega_{ws}}{\Omega_g}\right)$$

$$\begin{cases} \langle\theta\rangle\to\theta_0 = O(\ell_g)^2 \quad \text{when} \quad \omega\ll\omega_g=\dfrac{\kappa}{\rho^e c_p\theta_0}\approx\dfrac{2.10^{-5}}{(\ell_g)^2} \\ \dfrac{i\omega\rho^e c_p\langle\theta\rangle}{\kappa}\to 1 \quad \text{when} \quad \omega\gg\omega_g=\dfrac{\kappa}{\rho^e c_p\theta_0} \end{cases}$$

Quasi-isotherme

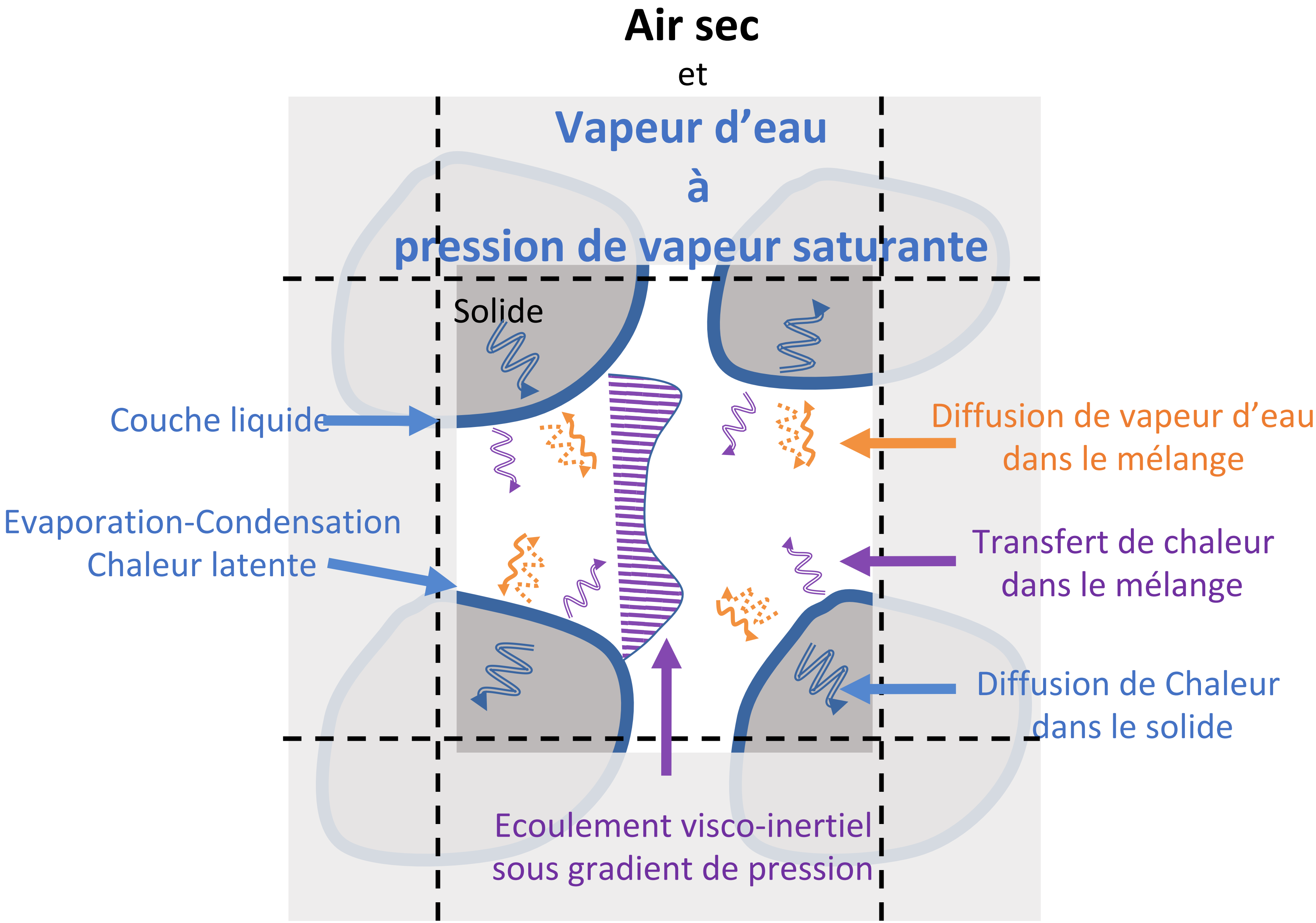
Quasi-adiabatique

$$\begin{cases} \langle\Theta\rangle\to\Theta_0 = O(\ell_s)^2 \quad \text{when} \quad \omega\ll\omega_s=\dfrac{\kappa_s}{(\rho^e c_p)_s\Theta_0}\approx\dfrac{1.510^{-6}}{(\ell_s)^2} \\ \dfrac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w}\to 1 \quad \text{when} \quad \omega\gg\omega_s=\dfrac{\kappa_s}{(\rho^e c_p)_s\Theta_0} \end{cases}$$

Compressibilité x 9

Effet négligeable

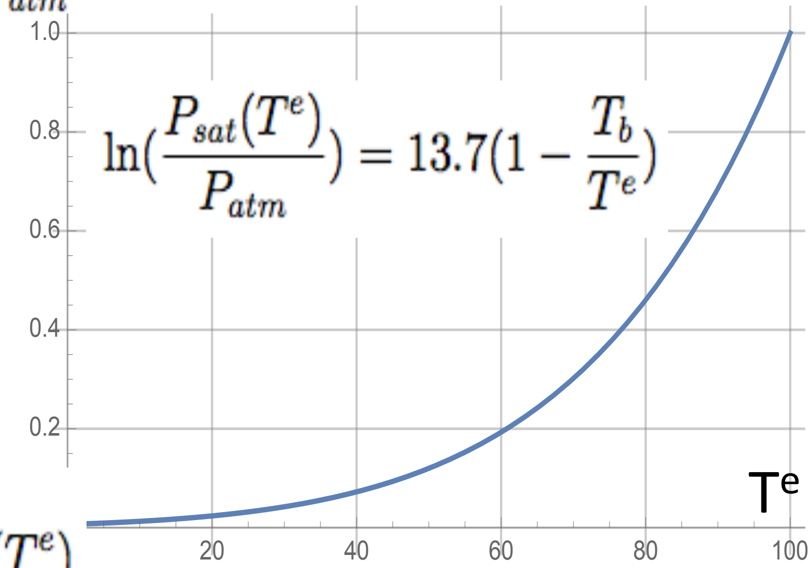
# Milieu à 100% Humidité



# Approche en Faible diffusion

**Pression de vapeur saturante $P_v^e$ à $T^e$ :**
**Rankine**

$$\frac{P_{sat}(T^e)}{P_{atm}} = f_R(T^e)$$




$$\ln\left(\frac{P_{sat}(T^e)}{P_{atm}}\right) = 13.7\left(1 - \frac{T_b}{T^e}\right)$$

**Equation d'éta** $\dfrac{p_v}{P_v^e} = \dfrac{\rho_v}{\rho_v^e} + \dfrac{\tau}{T^e}$

**Ordre domina** $\dfrac{p_v^{(0)}}{P_v^e} - \dfrac{P}{P^e} = \dfrac{\rho_v^{(0)}}{\rho_v^e} - \dfrac{\rho^{(0)}}{\rho^e}$

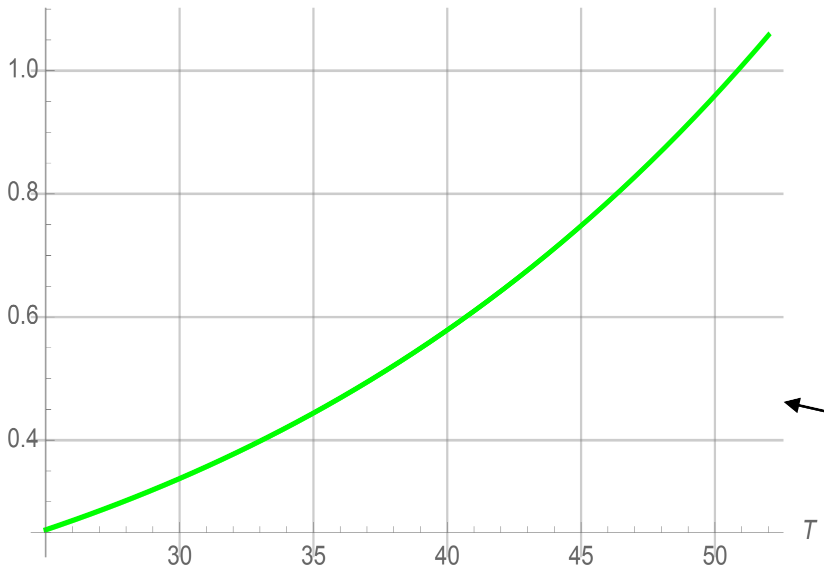
**Faible diffusior** $\dfrac{\rho_v^{(0)}}{\rho^{(0)}} = \dfrac{\rho_v^e}{\rho^e}$ donc $\dfrac{p_v^{(0)}}{P} = \dfrac{P_{sat}^e}{P^e} = \dfrac{P_v(\mathbf{x})}{P(\mathbf{x})} = f_R(T^e)$

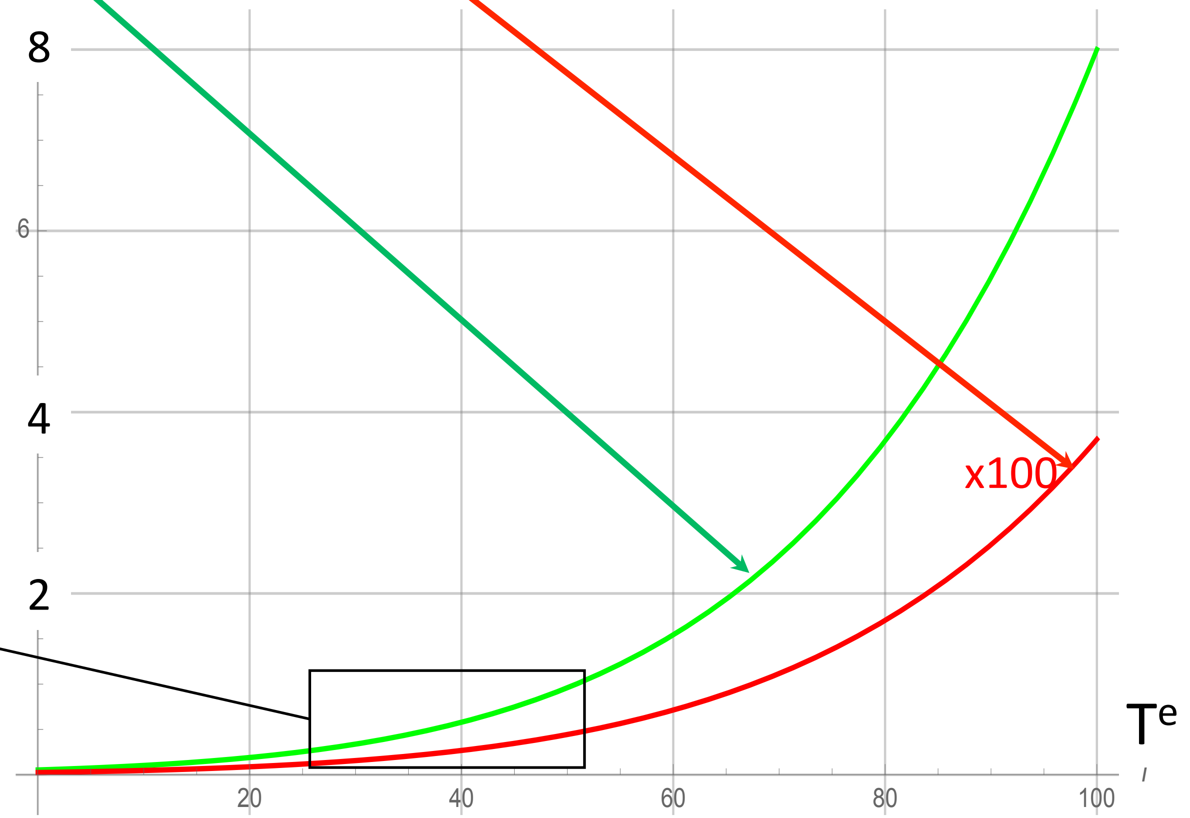
**Clapeyron sur** $\tau^{(0)} = T_\Gamma(\mathbf{x}) = P_v(\mathbf{x})\dfrac{T^e}{\mathcal{L}\rho_v^e} = \boxed{f_R(T^e)}P(\mathbf{x})\dfrac{T^e}{\mathcal{L}\rho_v^e}$ on $\Gamma$

# Compressibilité effective à $T^e$ et 100% Humidité

$$\mathrm{div}_x(\mathbf{V}) = -i\omega\frac{P}{P^e}\left(1-\boxed{\frac{f_R(T^e)P^e}{\mathcal{L}\rho_v^e}}\left(1+\left(\frac{\mathcal{L}\rho_v^e}{\rho^e c_p T^e f_R(T^e)}-1\right)\frac{i\omega\rho^e c_p\langle\theta\rangle}{\kappa}\right)\right)$$
$$-i\omega\frac{P}{P^e}\left(\boxed{\frac{f_R(T^e)P^eT^e\langle\rho c\rangle_{ws}}{(\mathcal{L}\rho_v^e)^2}}\left(1-\frac{i\omega\langle\rho c\Theta\rangle_{ws}}{\kappa_w}\right)\frac{\Omega_{ws}}{\Omega_g}\right)$$

**Effet de changement de phase selon $T^e$**



# Conclusion

**Effet notable à forte température**

**Nette dépendence frequentielle**

**Dissipation additionelle**

**Rôle du solide**

**Paramètres adimensionels du phénomène**

**Confrontation expérientale ?**

**Intégrer la diffusion vapeur-air ?**

# Quelques références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